Dear combinatorialists,
It is my great pleasure to welcome you at Heidelberg University for the 40th Colloquium on Combinatorics. The Colloquium on Combinatorics was established in 1981 and has since been held annually (with the exceptions in 2005, 2020, and 2021) in eight cities throughout Germany. This conference is today an established conference in Germany covering all areas of Combinatorics and Discrete Mathematics in a broad sense, including combinatorial aspects in Algebra, Geometry, Optimization and Computer Science.
This year there are 65 participants. The program includes 38 contributed talks, organised in up to four parallel sessions, and four invited talks on a broad range of combinatorial topics.
Please note that there are 25 -minute slots allocated for the contributed talks, which includes 20 minutes for the presentation, two minutes for discussion, and three minutes for room change.

Enjoy the conference!

Felix Joos

All talks will be in the Mathematikon (Im Neuenheimer Feld 205, 69120 Heidelberg)

| Invited talks | $:$ | Hörsaal |
| :--- | :--- | :--- |
| Contributed talks | $:$ | Hörsaal, Seminar Rooms A, B, C |
| Coffee and snacks | $:$ | Foyer (downstairs) |
| Registration desk | $:$ | Foyer (ground floor) |

The registration desk is open on Friday from 8:00 to 9:00 and on Saturday from 8:40 to 9:00.

The dinner will take place at the restaurant Kulturbrauerei (Leyergasse 6) on Friday at 19:00. The restaurant can be reached by a 50 -minute-walk or by bus and 9 minutes walk.

To go by bus, take line 31 (bound to Universitätsplatz) until Marstallstraße followed by a walk through the old town.

The closest bus and tram stop is Bunsengymnasium south of the Mathematikon.

There will be an informal dinner on Thursday evening at Café Merlin (Bergheimer Str. 85) starting at 18:00. Feel also free to join later once you arrive in Heidelberg.

A visit to the Schloss Heidelberg outside of the conference program is highly recommended.

## Thursday, 12 October 2023

18:00 and later Informal dinner at Café Merlin (Bergheimer Str. 85, Heidelberg)

## Friday, 13 October 2023

| 08:00-09:00 | Registration |
| :--- | :--- |
| 08:50-09:00 | Opening |
| 09:00-10:00 | Daniel Král' <br>  <br> "Quasirandom combinatorial structures " <br> 10:00-10:30 |
| 10:30-12:10 | Coffee break |
| 12:15-13:30 | Lunch |
| 14:00-15:00 | Vera Traub |
|  | "Approximation Algorithms for Connectivity Augmentation" |
| 15:00-15:30 | Coffee break |
| $\mathbf{1 5 : 3 0 - 1 7 : 1 0}$ | Parallel sessions |
| $\mathbf{1 9 : 0 0}$ | Dinner at Kulturbrauerei (Leyergasse 6, Heidelberg) |

## Saturday, 14 October 2023

| 09:00-10:00 | Johannes Carmesin <br>  <br> 10:00-10:30 |
| :--- | :--- |
| "Angry theorems and decompositions of 3-connected graphs" |  |
| 10:30-11:20 | Parallel sessions |
| $\mathbf{1 1 : 3 0 - 1 2 : 3 0}$ | Pascal Schweitzer <br> "Interactions between Graph Structure and Graph Automorphisms" <br> $\mathbf{1 2 : 3 0 - 1 3 : 3 0}$ |
| Lunch and Farewell |  |

## Colloquium on Combinatorics - 13/14 October 2023 <br> Discrete Mathematics - Heidelberg University

## Detailed program on Friday, 13 October 2023

| Time | Section I Hörsaal | Section II <br> Seminar Room: A | Section III <br> Seminar Room: B | Section IV Seminar Room: C |
| :---: | :---: | :---: | :---: | :---: |
| 08:50-09:00 | Opening Hörsaal |  |  |  |
| 09:00-10:00 | Daniel Král' Quasirandom combinatorial structures |  |  | Hörsaal |
| 10:00-10:30 | Coffee break |  |  |  |
| 10:30-10:50 | M. Axenovich $\quad 1$ Extremal functions in a hypercube | S. Brenner <br> Tuple regularity and $k$-ultrahomogeneity for finite groups | S. Cambie 3 <br> The average solution of a TSP instance in a graph | S. Albrechtsen A Menger-type theorem for two induced paths |
| 10:55-11:15 | R. Martin <br> Counting cycles in planar graphs | A. Polujan <br> Value distributions of perfect nonlinear functions | S. Guzmán-Pro <br> Forbidden Tournaments and the Orientation Completion Problem | F. Reich Menger's Theorem in bidirected graphs |
| 11:20-11:40 | D. Liu <br> The Typical Structure of Sets With Small Doubling | T. Kalayci <br> Bent partitions, strongly regular graphs, and association schemes | N. Jedličková <br> Min orderings and list homomorphism dichotomies for signed graphs | T. Hofmann <br> Uniformly connected graphs |
| 11:45-12:05 | M. A. Yildiz <br> Cycle Partitions in Dense Regular Digraphs and Oriented Graphs | T. Karam <br> Characterising qualitative separations between the ranks of tensors arising from partitions | C. Deppe <br> Non-Adaptive and Adaptive Two-Sided Search with Fast Objects | R. Jacobs Canonical graph decompositions via coverings |
| 12:15-13:30 | Lunch |  |  |  |

## Colloquium on Combinatorics - 13/14 October 2023 Discrete Mathematics - Heidelberg University

## Detailed program on Friday, 13 October 2023

| Time | Section I Hörsaal | Section II Seminar Room: A | Section III <br> Seminar Room: B | Section IV <br> Seminar Room: C |
| :---: | :---: | :---: | :---: | :---: |
| 14:00-15:00 | Vera Traub Approximation Algorithms for Connectivity Augmentation Hörsaal |  |  |  |
| 15:00-15:30 | Coffee break |  |  |  |
| 15:30-15:50 | A. Espuny Díaz <br> The threshold for spanning balanced binary trees in random geometric graphs | E. Ghorbani 18 Maximum diameter and minimum algebraic connectivity | J. Bok 19 List covering of regular multigraphs with semi-edges | T. Krill $\quad 20$The number of <br> topological types of trees |
| 15:55-16:15 | T. Fischer $\quad \mathbf{2 1}$ Fractional vs. Expectation Threshold | G. Rattan 22 <br> The Matrix Algebra of Graph Homomorphisms | L. Chidiac $\quad 23$ Positroids are 3-colorable | J. Rauch 24Sparse Vertex Cutsets and <br> the Maximum Degree |
| 16:20-16:40 | A. Allin $\mathbf{2 5}$ <br> Randomly perturbing  <br> graphs with a given  <br> degree sequence:  <br> Hamiltonicity and <br> pancyclicity  <br>   | D. Frettlöh <br> Counting tiles in <br> substitution tilings | S. Keip $\quad \mathbf{2 7}$ The Varchenko Determinant for Complexes of Oriented Matroids | J. Jooken 28 <br> Few hamiltonian cycles in ( $k, l$ )-regular graphs |
| 16:45-17:05 | R. Lang $\mathbf{2 9}$ <br> Resilience for loose  <br> Hamilton cycles  | K. Vorob'ev 30 <br> On existence of extended perfect codes | W. Hochstättler 31 <br> Hyperplane sweeps in Euclidean oriented matroids | I. Allie <br> Resistance, oddness and colouring defect of snarks |
| 19:00 | Conference Dinner |  |  |  |

## Colloquium on Combinatorics - 13/14 October 2023

 Discrete Mathematics - Heidelberg UniversityDetailed program on Saturday, 14 October 2023

| Time | Section I <br> Hörsaal | Section II <br> Seminar Room: A | Section III <br> Seminar Room: B | Section IV <br> Seminar Roo | m: C |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 09:00-10:00 | Johannes Carmesin Angry theorems and decompositions of 3-connected graphs |  |  |  | Hörsaal |
| 10:00-10:30 | Coffee break |  |  |  |  |
| 10:30-10:50 | None | M. Lichter 33 <br> The Iteration Number of the Weisfeiler-Leman Algorithm | A. Joó 34 <br> The Lovász-Cherkassky theorem in infinite graphs | D. Mattiolo On $d$-dimensional nowhere-zero $r$-flo a graph |  |
| 10:55-11:15 | None | D. Rautenbach Efficiently finding low-sum copies of spanning forests in zero-sum complete graphs via conditional expectation | M. Pitz <br> Improved bounds for Nash-Williams's orientation conjecture in infinite graphs | J. Renders <br> On the Frank Num Nowhere-Zero Flo Graphs |  |
| 11:30-12:30 | Pascal Schweitzer Interactions between Graph Structure and Graph Automorphisms |  |  |  | Hörsaal |
| 12:30-13:30 | Lunch and Farewell |  |  |  |  |

## Invited talks

| Johannes Carmesin : Angry theorems and decompositions of 3-connected graphs |  |
| :--- | :--- |
| Daniel Král' | : Quasirandom combinatorial structures |
| Pascal Schweitzer | : Interactions between Graph Structure and Graph Automor- |
| phisms |  |

## Contributed talks

| Sandra Albrechtsen | : A Menger-type theorem for two induced paths |
| :--- | :--- |
| Imran Allie | $:$ Resistance, oddness and colouring defect of snarks |
| Alexander Allin | : Randomly perturbing graphs with a given degree sequence: |
|  | Hamiltonicity and pancyclicity |


| iilo Krill | The number of topological types of trees |
| :---: | :---: |
| Richard Lang | : Resilience for loose Hamilton cycles |
| Moritz Lichter | : The Iteration Number of the Weisfeiler-Leman Algorithm |
| Dingyuan Liu | : The Typical Structure of Sets With Small Doubling |
| Ryan Martin | : Counting cycles in planar graphs |
| Davide Mattiolo | : On $d$-dimensional nowhere-zero $r$-flows on a graph |
| Max Pitz | : Improved bounds for Nash-Williams's orientation conjecture in infinite graphs |
| Alexandr Poluja | : Value distributions of perfect nonlinear functions |
| Gaurav Rattan | : The Matrix Algebra of Graph Homomorphisms |
| Johannes Rauch | : Sparse Vertex Cutsets and the Maximum Degree |
| Dieter Rautenbach | : Efficiently finding low-sum copies of spanning forests in zero-sum complete graphs via conditional expectation |
| Florian Reich | : Menger's Theorem in bidirected graphs |
| Jarne Renders | : On the Frank Number and Nowhere-Zero Flows on Graphs |
| Konstantin Vorob' | On existence of extended perfect codes |
| Mehmet Akif Yild | : Cycle Partitions in Dense Regular Digraphs and Oriented Graphs |

# Friday, 13 Oct. 2023 - Time: 09:00-10:00 - Hörsaal 

## Quasirandom combinatorial structures

Daniel Král’ (Masaryk University )

A combinatorial structure is said to be quasirandom if it resembles a random structure in a certain robust sense. The notion of quasirandom graphs, developed in the work of Rödl, Thomason, Chung, Graham and Wilson in 1980s, is particularly robust as several different properties of truly random graphs, e.g., subgraph density, edge distribution and spectral properties, are satisfied by a large graph if and only if one of them is. We will discuss quasirandom properties of various combinatorial structures, in particular, directed graphs, permutations and Latin squares, and present recent results obtained using analytic tools of the theory of combinatorial limits.

# Friday, 13 Oct. 2023 - Time: 14:00-15:00 - Hörsaal 

## Approximation Algorithms for Connectivity Augmentation

Vera Traub (University of Bonn)

The goal of network design is to construct cheap networks that satisfy certain connectivity requirements. The famous algorithm by Jain [Combinatorica, 2001] provides a 2-approximation for a wide class of these problems. However, even for many very basic special cases nothing better is known.

One of the most elementary network design problems is the the weighted connectivity augmentation problem, which asks to increase the edge-connectivity of a graph in the cheapest possible way by adding edges from a given set. In this talk we present the first approximation algorithms for weighted connectivity augmentation that beat the longstanding approximation factor of 2 . We also discuss extensions and implications of our techniques to related network design problems such as the forest augmentation problem and the Steiner tree problem.

This talk is based on joint work with Fabrizio Grandoni, Afrouz Jabal Ameli, and Rico Zenklusen.

## Saturday, 14 Oct. 2023 - Time: 09:00-10:00 - Hörsaal

## Angry theorems and decompositions of 3-connected graphs

Johannes Carmesin (University of Birmingham)

We offer a new structural basis for the theory of 3-connected graphs, providing a unique decomposition of every such graph into parts that are either quasi 4-connected, wheels, or thickened $K_{3, m}$ 's. Our construction is explicit, canonical, and has the following applications: we obtain a new theorem characterising all Cayley graphs as either essentially 4-connected, cycles, or complete graphs on at most four vertices, and we provide an automatic proof of Tutte's wheel theorem, as well as follow-up works including new FPT-algorithms.

# Saturday, 14 Oct. 2023 - Time: 11:30-12:30 - Hörsaal 

## Interactions between Graph Structure and Graph Automorphisms

Pascal Schweitzer (TU Darmstadt)

Symmetries and structural considerations are recurring themes in graph theory. They may seem a priori unrelated. However, over the years numerous insights have shown that one can advance the other. My goal in this talk is to survey results that exemplify this synergetic behavior. For example, we will see how the existence or absence of particular graph symmetries, more precisely automorphisms, can force structural properties. In the other direction, we will also see that structural properties of a graph can influence the existence of symmetries. The graph structure thus impacts the group of automorphisms. Throughout the talk we will see old and new results that highlight, stress, and exploit this type of connection between the two themes from different angles.

As a recurring technique that relates structure and automorphisms, we will encounter the use of improper vertex colorings (meaning that adjacent vertices may have the same color). One perspective is that by using colors we can artificially break symmetries of a graph, thereby restricting the automorphisms. The other perspective is that the lack of symmetries can often be verified by basic algorithmic methods, in which colors carve out symmetries. Coherent configurations, a notion from algebraic graph theory, emerge naturally from these considerations.

At various times in the talk, I will also touch on complexity questions that arise in the context of automorphism groups of graphs and coloring techniques. This overview covers results from numerous collaborations over the years with many co-authors, with the ultimate aim of deepening our understanding of graph isomorphisms.

# Friday, 13 Oct. 2023 - Time: 10:30-10:50 

1 - Section I - HS - 10:30-10:50

## Extremal functions in a hypercube

Maria Axenovich (Karlsruhe Institute of Technology)

One of the central problems in graph theory is finding, for a given $r$-graph $H$, the extremal function $e x_{r}(n, H)$, that is the largest number of edges in an $n$-vertex $r$-graph that contains no isomorphic copy of $H$ as a subgraph. While determining the asymptotic behaviour of $e x_{r}(n, H)$ remains a challenge in general, we know exactly what graphs have positive Turán density. An analogous function ex $\left(Q_{n}, H\right)$, the largest number of edges in a subgraph of the $n$-dimensional hypercube $Q_{n}$ that contains no isomorphic copy of $H$, is much less understood. In particular, we even do not have any characterisation for graphs $H$ that have a positive hypercube Turán density. In this talk I will report on some recent progress on $e x\left(Q_{n}, H\right)$ and show connections between this function and other problems in extremal combinatorics.

2 - Section II - SR A - 10:30-10:50

## Tuple regularity and $k$-ultrahomogeneity for finite groups

Sofia Brenner (Technische Universität Darmstadt)

Studying finite groups from a combinatorial perspective leads to a demand for combinatorial symmetry and regularity measures. In this context, we develop the concepts of $k$-ultrahomogeneity and $\ell$-tuple regularity for finite groups. They are inspired by analogous concepts in graph theory and form natural generalizations of previously known properties such as ultrahomogeneity. In my talk, I will discuss both notions and present a classification result.

## The average solution of a TSP instance in a graph

Stijn Cambie (KU Leuven)

In many practical situations (mostly logistics- or routing-related), one wants to find the most efficient route visiting some places. For example, in a warehouse, one may want to pick the items from different spots that are necessary for a delivery. In this case, one needs to solve a so-called TSP (travelling salesman problem) instance. This is a well-studied topic, where in the abstract form, one has a (possibly weighted) graph and one needs to find the shortest walk visiting a certain list of vertices.

When constructing the network; city plan or some circuit design, one may aim that the logistics will be efficient in most of the cases. That is, one may want to know the graphs for which the TSP instance will have a small cost. As such, it is interesting to study the graphs that have extremal solutions for the average cost of the TSP instance.

In this talk, we give some definitions and ideas about the extremal graphs, and observe relations with other concepts.

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4 \text { - Section IV - SR C - 10:30-10:50 }
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## A Menger-type theorem for two induced paths

Sandra Albrechtsen (Universität Hamburg)

Given a graph $G$ and $X, Y \subseteq V(G)$, Menger's theorem states that either there exist $k$ pairwise disjoint $X-Y$ paths or there exists a set of $k-1$ vertices that intersects every $X-Y$ path. We believe that there is also a Menger-type theorem characterizing when a graph contains $k X-Y$ paths that are not just pairwise disjoint, but even pairwise far apart in $G$.

Such a characterization is unlikely to be exact, given that it is already NP-complete to decide whether there exist two $X-Y$ paths at distance at least 2 . But we conjecture the following.

Conjecture: There exists a constant $c$ such that for all graphs $G$ and $X, Y \subseteq V(G)$ and integers $d, k \geq 1$, either there exist $k$ disjoint $X-Y$ paths $P_{1}, \ldots, P_{k}$ such that $\operatorname{dist}_{G}\left(P_{i}, P_{j}\right) \geq d$ for all distinct $i, j$ or there exists a set $Z \subseteq V(G)$ of size at most $k-1$ such that $B_{G}(Z, c d)$ intersects every $X-Y$ path.

Here, $\operatorname{dist}_{G}\left(P_{i}, P_{j}\right)$ denotes the distance between $P_{i}$ and $P_{j}$, that is, the minimum length of a $V\left(P_{i}\right)-$ $V\left(P_{j}\right)$ path, and $B_{G}(Z, c d)$ is the set of all vertices in $G$ that have distance at most $c d$ from $Z$. We provide evidence for this conjecture by proving the case $k=2$.
This is joint work with Tony Huynh, Raphael W. Jacobs, Paul Knappe and Paul Wollan.

# Friday, 13 Oct. 2023 — Time: 10:55-11:15 

5 - Section I - HS - 10:55-11:15

## Counting cycles in planar graphs

Ryan Martin (Iowa State University)

Basic Turán theory asks how many edges a graph can have, given certain restrictions such as not having a large clique.

A more generalized Turán question asks how many copies of a fixed subgraph $H$ the graph can have, given certain restrictions. There has been a great deal of recent interest in the case where the restriction is planarity.

In this talk, we will discuss some of the general results in the field, primarily the asymptotic value of $\mathbf{N}_{\mathcal{P}}(n, H)$, which denotes the maximum number of copies of $H$ in an $n$-vertex planar graph. In particular, we will focus on the case where $H$ is a cycle. It was determined that $\mathbf{N}_{\mathcal{P}}\left(n, C_{2 m}\right)=$ $(n / m)^{m}+o\left(n^{m}\right)$ for small values of $m$ by Cox and Martin and resolved for all $m$ by Lv, Győri, He, Salia, Tompkins, and Zhu.

The case of $H=C_{2 m+1}$ is more difficult and it is conjectured that $\mathbf{N}_{\mathcal{P}}\left(n, C_{2 m+1}\right)=2 m(n / m)^{m}+$ $o\left(n^{m}\right)$. We will discuss recent progress on this problem, including verification of the conjecture in the case where $m=3$ and $m=4$ and a lemma which reduces the solution of this problem for any $m$ to a so-called "maximum likelihood" problem. The maximum likelihood problem is, in and of itself, an interesting question in random graph theory.

This is joint work with Emily Heath and Chris (Cox) Wells.
6 - Section II - SR A - 10:55-11:15

## Value distributions of perfect nonlinear functions

Alexandr Polujan (Otto von Guericke University Magdeburg)

Perfect nonlinear functions (also called bent functions) are in the most general sense mappings between two finite groups that are "as far away" from homomorphisms as possible [1]. They have wellknown connections to cryptography, coding theory, and design theory. In this talk, I shall discuss the value distributions of bent functions, i.e., the cardinalities of their image and preimage sets. It turns out that very strong conditions on the sizes of the preimage sets can be derived. Moreover, many well-known constructions of perfect nonlinear functions have in some sense an extremal value distribution. I shall also present some complete classification results for value distributions of perfect
nonlinear functions between specific groups, and connections between value distributions and other properties of perfect nonlinear functions. The talk is based on joint work [2] with Lukas Kölsch from the University of South Florida.

## Literatur

[1] Alexander Pott. Nonlinear functions in abelian groups and relative difference sets. Discret. Appl. Math. 138(1-2), 177-193 (2004).
[2] Lukas Kölsch and Alexandr Polujan. "Value distributions of perfect nonlinear functions." Accepted for publication in Combinatorica. arXiv preprint arXiv:2302.03121 (2023).

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7 \text { - Section III - SR B - 10:55-11:15 }
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## Forbidden Tournaments and the Orientation Completion Problem

Santiago Guzmán-Pro (TU Dresden)

For a fixed finite set of finite tournaments $\mathcal{F}$, the $\mathcal{F}$-free orientation problem asks whether a given finite undirected graph $G$ has an $\mathcal{F}$-free orientation, i.e., whether the edges of $G$ can be oriented so that the resulting oriented graph does not contain any tournament from $\mathcal{F}$ as an oriented (induced) subgraph. We prove that for every $\mathcal{F}$, this problem is in P or NP-complete. Specifically, we show that either the $\mathcal{F}$-free orientation problem can be reduced (in polynomial-time) to a system of Boolean linear equations, or the $\mathcal{F}$-free orientation problem is NP-complete.

We reduce the classification task to a complete complexity classification of the orientation completion problem for $\mathcal{F}$, which is the variant of the problem above where the input is a directed graph instead of an undirected graph, introduced by Bang-Jensen, Huang, and Zhu (2017). Our proof uses results from the theory of constraint satisfaction, and a result of Agarwal and Kompatscher (2018) about infinite permutation groups and transformation monoids.

This is joint work with Manuel Bodirsky.

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8 \text { - Section IV - SR C - 10:55-11:15 }
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## Menger's Theorem in bidirected graphs

Florian Reich (Universität Hamburg)

Bidirected graphs are a generalisation of directed graphs that arises in the study of undirected graphs with perfect matchings. Menger's famous theorem - the minimum size of a set separating two vertex sets $X$ and $Y$ is the same as the maximum number of disjoint paths connecting them - is generally not true in bidirected graphs. We introduce a sufficient condition for $X$ and $Y$ which yields a version of Menger's Theorem in bidirected graphs that in particular implies its directed counterpart. This is joint work with N. Bowler, E. Ghorbani, F. Gut and R. W. Jacobs.

# Friday, 13 Oct. 2023 - Time: 11:20-11:40 

## $9-$ Section I - HS - 11:20-11:40

# The Typical Structure of Sets With Small Doubling 

Dingyuan Liu (Karlsruhe Institute of Technology)

We study the sets with small doubling using graph containers. Let $n, s$ be sufficiently large positive integers and $K=o(s)$, it is expected that every s-element subset $A \subseteq[n]$ with $|A+A| \leq K s$ should have a structure that is close to an arithmetic progression of length $O(K s)$. However, despite a long line of research, one is still far from verifying the above intuition for $K \gg O(1)$. In this joint work with Letícia Mattos and Tibor Szabó, we prove a tight upper bound on the number of $s$-element subsets $A \subseteq[n]$ with $|A+A| \leq K s$ when $K=o\left(\frac{s}{(\log n)^{2}}\right)$, and that almost all such sets are close to arithmetic progressions of length $\frac{K s}{2}+o(K s)$. This extends the range of $K$ in the previous work by Campos and partially confirms an earlier conjecture of Alon, Balogh, Morris, and Samotij.

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10 \text { - Section II - SR A - 11:20-11:40 }
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## Bent partitions, strongly regular graphs, and association schemes

TEKGÜL KALAYCI (Sabanci University)

Recently, the concept of bent partitions was introduced by Anbar and Meidl, 2022, which are partitions of elementary abelian groups having similar properties as spreads. A partition $\Omega$ of $\mathbb{V}_{2 m}^{(p)}$ into an $m$-dimensional subspace $U$, and subsets $A_{1}, \ldots, A_{K}$, is called a bent partition of depth $K$ if every function from $\mathbb{V}_{2 m}^{(p)}$ to $\mathbb{F}_{p}$ with the following properties, is a bent function: Every $c \in \mathbb{F}_{p}$ has precisely $K / p$ of the sets $A_{i}$ in its preimage set $f^{-1}(c)=\left\{x \in \mathbb{V}_{2 m}^{(p)}: f(x)=c\right\}$ and $f$ is constant on $U$. From (pre)semifields whose duals satisfy a linearity property over a subfield of $\mathbb{F}_{p^{m}}$, we construct a large class of bent partitions of $\mathbb{F}_{p^{m}} \times \mathbb{F}_{p^{m}}$, which we call generalized semifield spreads. We show that generalized semifield spreads do not arise from any spread and one obtains not only $p$-ary and vectorial bent functions, but also bent functions into finite abelian groups of certain orders from generalized semifield spreads. We then show that any union of the sets in generalized semifield spreads are partial difference sets. Furthermore, such a partition induces an edge decomposition of the complete graph with the vertex set $\mathbb{F}_{p^{m}} \times \mathbb{F}_{p^{m}}$ into strongly regular graphs, for which any union of the graphs is again strongly regular. This enables us to construct $\left(p^{k}+1\right)$-class amorphic association schemes on $\mathbb{F}_{p^{m}} \times \mathbb{F}_{p^{m}}$ from (pre)semifields, where $k$ is a divisor of $m$ with special properties. This is joint work with Nurdagül Anbar and Wilfried Meidl.

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# Min orderings and list homomorphism dichotomies for signed graphs 

NIKOLA JEDLIČKovÁ (Charles University)

We investigate the complexity of list homomorphism problems for signed graphs. The CSP dichotomy conjecture has been recently established, but a number of other dichotomy questions remain open, including the dichotomy classification of list homomorphism problems for signed graphs. The list homomorphism problem for a signed graph $\widehat{H}$ asks whether an input signed graph $\widehat{G}$ with lists $L(v) \subseteq V(\widehat{H}), v \in V(\widehat{G})$, admits a homomorphism $f$ to $\widehat{H}$ with all $f(v) \in L(v), v \in V(\widehat{G})$. Usually, a dichotomy classification is easier to obtain for list homomorphisms than for homomorphisms, but for signed graphs it is not the case. The complexity classification for list homomorphism problem appears to be difficult, and no structural classification conjecture has arisen. In this talk, I will show a structural classification in the special case of weakly balanced irreflexive and reflexive signed graphs (previously conjectured by Kim and Siggers). This generalizes previous results on weakly balanced signed trees, and weakly balanced separable signed graphs. The proof of irreflexive case depends on first deriving a theorem on extensions of min orderings of (unsigned) bipartite graphs, which is interesting on its own. In both cases the dichotomy classification depends on a result linking the absence of certain structures called chains and invertible pairs to the existence of a so-called special min ordering.

This is joint work with Jan Bok, Richard Brewster, Pavol Hell and Arash Rafiey.

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12 \text { - Section IV - SR C - 11:20-11:40 }
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## Uniformly connected graphs

Tobias Hofmann (TU Berlin)

A graph $G$ on at least $k+1$ vertices is called uniformly $k$-connected if any two vertices of $G$ can be connected by $k$ and not more than $k$ internally vertex disjoint paths. In this talk, we are concerned with the structure of uniformly $k$-connected and uniformly $k$-edge-connected graphs, which are defined analogously. Whereas both types have previously been studied independent of each other, we analyze relations between those two classes. We prove that any uniformly $k$-connected graph is also uniformly $k$-edge-connected for $k \leq 3$ and demonstrate that this is not the case for $k>3$. Another central contribution is a constructive characterization of uniformly 3 -connected graphs that complements Tutte's Wheel Theorem. For example, this result allows to prove a tight bound on the number of vertices of minimum degree in uniformly 3 -connected graphs.

# Friday, 13 Oct. 2023 - Time: 11:45-12:05 

## 13 - Section I - HS - 11:45-12:05

## Cycle Partitions in Dense Regular Digraphs and Oriented Graphs

## Mehmet Akif Yildiz (University of Amsterdam)

A conjecture of Jackson from 1981 states that every $d$-regular oriented graph on $n$ vertices with $n \leq 4 d+1$ is Hamiltonian. We prove this conjecture for sufficiently large $n$ by showing a more general result about cycle partitions in dense regular digraphs and oriented graphs which states that for all $\alpha>0$, there exists $n_{0}=n_{0}(\alpha)$ such that every $d$-regular digraph on $n \geq n_{0}$ vertices with $d \geq \alpha n$ can be covered by at most $n /(q d+1)$ vertex-disjoint cycles where $q=2$ if $G$ is oriented and $q=1$ otherwise.

This is joint work with Allan Lo (University of Birmingham) and Viresh Patel (Queen Mary University of London).

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14 \text { - Section II - SR A - 11:45-12:05 }
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## Characterising qualitative separations between the ranks of tensors arising from partitions

Thomas Karam (University of Oxford)

The notion of rank on matrices has been extended in several ways to notions of rank on higherdimensional tensors. Well-known among these generalisations are in particular the tensor rank, the slice rank, and the partition rank. The definitions of many of these notions (and in particular the three which we have previously mentioned) can be unified, as follows. For $d \geq 2$ an integer and R a nonempty set of partitions of $[d]$ we say that an order- $d$ tensor $T$ has R-rank at most 1 if we can write the entries of $T$ as a product of functions

$$
T\left(x_{1}, \ldots, x_{d}\right)=\prod_{I \in P} a_{I}\left(x_{i}: i \in I\right)
$$

for some partition $P \in \mathrm{R}$. We then define the R-rank of $T$ as the smallest nonnegative integer $k$ such that we can write $T$ as a sum of $k$ tensors with R-rank at most 1.

We will study the relations between these notions of rank, focusing on and answering the following question: given positive integers $d, s$ and some sets of partitions $\mathrm{R}_{1}, \ldots, \mathrm{R}_{s}$ of $[d]$, for which sets R of partitions of $[d]$ is it true that if an order- $d$ tensor has bounded $\mathrm{R}_{i}$-rank for each $i=1, \ldots, s$, then
it must have bounded R-rank ? On the way to our proof of the harder implication in the case $s=1$ we will make progress on a related question of Naslund on the ranks of products of diagonal tensors. Then, in the case $s=2$ (which suffices to conclude for all $s$ ) the converse implication will involve its own difficulties.

## 15 - Section III - SR B - 11:45-12:05

## Non-Adaptive and Adaptive Two-Sided Search with Fast Objects

Christian Deppe (Technical University of Munich)

In 1946 Koopman introduced a two-sided search model. In this model, a searched object is active and can move at most one step after each test. We analyze the model of a combinatorial two-sided search by allowing more moves of the searched object after each test. We give strategies and show that they are optimal. We consider adaptive and non-adaptive strategies. We show the surprising result that with the combinatorial two-sided search on a path graph, the optimal non-adaptive search needs the same number of questions as the corresponding adaptive strategy does. The strategy obtained can also be used as encoding strategy to sent the position of a moving element through a transmission channel.

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16 \text { - Section IV - SR C - 11:45-12:05 }
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## Canonical graph decompositions via coverings

Raphael W. Jacobs (Universität Hamburg)

We present a canonical way to decompose finite graphs into highly connected local parts. The decomposition depends only on an integer parameter whose choice sets the intended degree of locality. The global structure of the graph, as determined by the relative position of these parts, is described by a coarser model. This is a simpler graph determined entirely by the decomposition, not imposed.

The model and decomposition are obtained as projections of the tangle-tree structure of a covering of the given graph that reflects its local structure while unfolding its global structure. In this way, the tangle theory from graph minors is brought to bear canonically on arbitrary graphs, which need not be tree-like.

This is joint work with Reinhard Diestel, Paul Knappe and Jan Kurkofka.

# Friday, 13 Oct. 2023 — Time: 15:30-15:50 

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17 \text { - Section I - HS - 15:30-15:50 }
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# The threshold for spanning balanced binary trees in random geometric graphs 

Alberto Espuny DíaZ (Universität Heidelberg)

Consider the following model of random graphs: a total of $n$ vertices are assigned to uniformly random positions on the unit square, independently of each other, and any two vertices are then joined by an edge if the distance between their positions is less than a given parameter $r$. This is called the random geometric graph $G(n, r)$ and, similarly to the binomial random graph $G(n, p)$, increasing properties exhibit thresholds (with respect to the parameter $r$ ) which we wish to understand. The behaviour of random geometric graphs, however, is very different from the behaviour of $G(n, p)$.

In this talk, I will highlight some of these differences and eventually focus on the case of balanced $s$-ary trees, for which we have established the threshold. This is based on joint work with Lyuben Lichev, Dieter Mitsche and Alexandra Wesolek.

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18 \text { - Section II - SR A - 15:30-15:50 }
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## Maximum diameter and minimum algebraic connectivity

Ebrahim Ghorbni (K.N.T. University of Technology and University of Hamburg)

In the literature, it has been empirically observed that graphs with small algebraic connectivity $\mu$ (the second smallest eigenvalue of the Laplacian matrix), tend to have a large diameter. In this context, Guiduli (1997) posed the question of whether cubic graphs with the maximum diameter have algebraic connectivity smaller than all others. Motivated by these, we investigate the interplay between the graphs with maximum diameter and those with minimum algebraic connectivity. We show that the answer to Guiduli problem in its general form, that is for $d$-regular graphs for every $d \geq 3$ is negative. Furthermore, we attempt to formulate the problem asymptotically. For the cases of $d=3$ and $d=4$, we prove that $d$-regular graphs with asymptotically maximum diameter also have asymptotically smallest $\mu$. However, this relationship does not hold for $d$-regular graphs with $d \geq 5$ or graphs with $\delta=d$ for $d \geq 4$. We put forth a conjecture that $d$-regular graphs (or graphs with $\delta=d$ ) having asymptotically smallest $\mu$ should exhibit asymptotically maximum diameter. The aformentioned results rely heavily on our understanding of the structure as well as optimal estimation of the algebraic connectivity of graphs with asymptotically maximum diameter. As a by-product, we establish that for odd $d$, a conjecture by Guiduli and Mohar (1997) regarding the structure of $d$-regular graphs with minimum algebraic connectivity implies the Aldous-Fill conjecture. This conjecture states that the maximum relaxation time for a random walk on a connected regular graph with $n$ vertices is
$(1+o(1)) \frac{3 n^{2}}{2 \pi^{2}}$. We also propose another conjecture on the structure of $d$-regular graphs with minimum $\mu$, and demonstrate that it implies the Aldous-Fill conjecture for even $d$.

This talk is based on a joint work with Maryam Abdi.

## List covering of regular multigraphs with semi-edges

Jan Bok (University of Clermont Auvergne)

A graph covering projection, also known as a locally bijective homomorphism, is a mapping between vertices and edges of two graphs which preserves incidences and which is a local bijection on the edge-neighbourhood of every vertex. This notion stems from topological graph theory, but has also found applications in combinatorics and theoretical computer science.

For a fixed target graph $H$, the $H$-Cover problem asks if an input graph $G$ allows a graph covering projection onto $H$. Despite the fact that the quest for characterizing the computational complexity of $H$-Cover had been started more than 30 years ago, only a handful of general results have been known so far.

In line with the recent development in topological graph theory, we are considering undirected graphs that are allowed to contain multiple edges, loops, and semi-edges. I will briefly summarise our ongoing endeavour in this direction and then focus on the list version of the problem, called LIST- $H$-COVER, where the vertices and edges of the input graph come with lists of admissible targets. I will conclude with the NP-co/polytime dichotomy for the computational complexity of LIST- $H$-COVER for cubic graphs.

Based on joint work with Jiří Fiala, Nikola Jedličková, Jan Kratochvíl, and Paweł Rzażewski.

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20 \text { - Section IV - SR C - 15:30-15:50 }
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## The number of topological types of trees

Thilo Krill (Universität Hamburg)

Two graphs are of the same topological type if they can be mutually embedded into each other topologically. We show that there are exactly $\aleph_{1}$ distinct topological types of countable trees. In general, for any infinite cardinal $\kappa$ there are exactly $\kappa^{+}$distinct topological types of trees of size $\kappa$. This solves a problem of van der Holst from 2005. The talk is based on joint work with Max Pitz.

# Friday, 13 Oct. 2023 - Time: 15:55-16:15 

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21 \text { - Section I - HS - 15:55-16:15 }
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## Fractional vs. Expectation Threshold

Thomas Fischer (TU Ilmenau)

A conjecture of Talagrand (2010) states that the so-called expectation and fractional expectation thresholds are always within at most some constant factor from each other. Expectation (resp. fractional expectation) threshold $q$ (resp. $q_{f}$ ) for an increasing nontrivial class $\mathcal{F} \subseteq 2^{X}$ allows to locate the threshold for $\mathcal{F}$ within a logarithmic factor (these are important breakthrough results of Park and Pham (2022), resp. Frankston, Kahn, Narayanan and Park (2019)). I will give an overview of the problem and discuss some known special cases.

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22 \text { - Section II - SR A - 15:55-16:15 }
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## The Matrix Algebra of Graph Homomorphisms

Gaurav Rattan (TU Darmstadt)

Lovász (1967) showed that two graphs $G$ and $H$ are isomorphic if and only if they are homomorphism indistinguishable over the class of all graphs, i.e. for every graph $F$, the number of homomorphisms from $F$ to $G$ equals the number of homomorphisms from $F$ to $H$. This remarkable result forms the starting point for a versatile and fruitful connection between graph homomorphisms and matrix algebras.

Recently, homomorphism indistinguishability over restricted classes of graphs, such as planar graphs or graphs of bounded treewidth, has emerged as a surprisingly powerful framework for capturing diverse relaxations of graph isomorphism. In this talk, we present a unified algebraic framework for such results by unraveling the representation-theoretic properties of graph homomorphisms.

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23 \text { - Section III - SR B - 15:55-16:15 }
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## Positroids are 3-colorable

Lamar Chidiac (Fernuniversität in Hagen)

Hadwiger's conjecture states that any graph which is not $k$-colorable must have a $K_{k+1}$ minor. Using the definition of the chromatic number of oriented matroid introduced by J. Nesetril, R. Nickel, and W. Hochstättler, Hochstättler presented a generalisation of Hadwiger's conjucture to regular oriented matroids and proved that it is equivalent to Tutte's 4 -flow and 5 -flow conjectures when $\mathrm{k}=4$ and $\mathrm{k}=5$ respectively, and that it is equivalent to Hadwiger's conjecture for $k \geq 6$. The case $k=3$ remains
the only non-trivial case. For general oriented matroid even the case $k=3$ remains open. Goddyn, Hochstättler and Neudauer introduced the class of generalised series parallel (GSP) oriented matroids, which is an $M\left(K_{4}\right)$-free class and proved it is 3-colorable. If all $M\left(K_{4}\right)$-free oriented matroids are GSP, then Hadwiger's conjecture would hold for oriented matroids in the case of $k=3$. We prove that positroids are GSP and therefore 3-colorable.

24 - Section IV - SR C - 15:55-16:15

## Sparse Vertex Cutsets and the Maximum Degree

## Johannes Rauch (Ulm University)

We propose the following question: Is there a function $f$ with $f(\Delta) \rightarrow \infty$ as $\Delta \rightarrow \infty$ such that each sufficiently large connected graph of maximum degree at most $\Delta$ admits a vertex cutset $S$ that induces a graph $G[S]$ of maximum degree at most $\Delta-f(\Delta)$ ? We provide a construction that shows $f(\Delta)=\mathcal{O}(\sqrt{\Delta} \log \Delta)$. We show that every graph $G$ of maximum degree $\Delta$ and sufficiently large order has a vertex cutset $S$ of order at most $\Delta$ that induces a subgraph $G[S]$ of maximum degree at most $\Delta-3$, which beats the trivial bound by one. For $\Delta \in\{4,5\}$, we refine this result by considering also the average degree of $G[S]$. If $G$ has no $K_{r, r}$ subgraph, then we show the existence of a vertex cutset that induces a subgraph of maximum degree at most $\left(1-\binom{r}{2}^{-1}\right) \Delta+\mathcal{O}(1)$.

# Friday, 13 Oct. 2023 - Time: 16:20-16:40 

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25 \text { - Section I - HS - 16:20-16:40 }
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## Randomly perturbing graphs with a given degree sequence: Hamiltonicity and pancyclicity

Alexander Allin (Technische Universität Ilmenau)

We consider Hamilton cycles in randomly perturbed graphs, that is, graphs obtained as the union of a deterministic graph $H$ and a random graph. For this random pertubation we consider both the binomial random graph $G(n, p)$ and the geometric random graph $G(n, r)$. While most research into randomly perturbed graphs assumes a minimum degree condition on $H$, here we consider conditions on its degree sequence. Under the assumption of a degree sequence of $H$ which is comparable with the classical condition of Chvátal (dependent on a parameter $\alpha$ analogous to the minimum degree condition in typical results in the area), we prove that there exists some constant $C=C(\alpha)$ such that taking $r=\sqrt{C / n}$ suffices to a.a.s. obtain a Hamilton cycle in $H \cup G(n, r)$. Moreover we show that these conditions also ensure that a.a.s. $H \cup G(n, r)$ is pancyclic.

Under the same conditions on $H$ we further prove that there is a constant $K=K(\alpha)$ such that $p=K / n$ guarantee that $H \cup G(n, p)$ is a.a.s. pancyclic. Our results are best possible both in terms of the degree sequence condition and the asymptotic value of $p$ and $r$, and extend the known results about Hamiltonicity in randomly perturbed graphs.

This is joint work with Alberto Espuny Díaz.

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26 \text { - Section II - SR A - 16:20-16:40 }
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## Counting tiles in substitution tilings

Dirk Frettlöh (Univ. Bielefeld)

For classical substitution tilings on finite sets of prototiles, the order of growth of the number of tiles in a large supertile is governed by the Perron-Frobenius eigenvalue. The order of the second term in the counting function can be either exponential or polynomial times exponential where the exponent comes from the second largest eigenvalue of the substitution matrix. In this talk we will discuss that for substitutions on infinite sets of prototiles the second term may behave differently. Particularly, we will show that it may behave as the sequence of Catalan numbers or as ratio of exponent and half-integer power for a certain family of substitutions. The talk is based on a joint work with Alexey Garber (UT Rio Grande Valley) and Neil Mañibo (Uni Bielefeld).

# The Varchenko Determinant for Complexes of Oriented Matroids 

## Sophia KeIp (FernUniversität in Hagen)

Let $\mathcal{A}$ be a real hyperplane arrangement and $L(\mathcal{A})$ the geometric lattice formed by the intersections of hyperplanes in $\mathcal{A}$. We call the fulldimensional cells of $\mathcal{A}$ topes. The Varchenko Matrix ist defined by $\mathcal{V}_{i j}=\prod_{e \in S\left(T_{i}, T_{j}\right)} w_{e}$, where the $w_{e}$ are weights on the hyperplanes $H_{e}$ of the arrangement and $S\left(T_{i}, T_{j}\right)$ is the set of hyperplanes that have to be crossed on a shortest path from a tope $T_{i}$ to a tope $T_{j}$. Varchenko [1] gave an elegant factorization of the determinant of that matrix, considering the weights as variables:

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\begin{equation*}
\operatorname{det}(\mathcal{V})=\prod_{F \in L(\mathcal{A})}\left(1-w_{F}^{2}\right)^{m_{F}} \tag{1}
\end{equation*}
$$

where $w_{F}=\prod_{F \subset H_{e}} w_{e}$ and $m_{F}$ are positive integers depending only on the geometric lattice $L(\mathcal{A})$. We generalize this theorem for a combinatorial structure called complexes of oriented matroids. They can be described by only two axioms which capture local symmetry and local convexity and are a generalization of oriented matroids. In this talk we will see how the Varchenko Matrix generalizes to complexes of oriented matroids and will give the general idea of the proof for the nice factorization formula.

This is joint work with Winfried Hochstättler (FernUniversität in Hagen) and Kolja Knauer (Universitat de Barcelona).

## Literatur

[1] A. Varchenko, Bilinear form of real configuration of hyperplanes, Adv. Math. 97 (1993) 110144.

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28 \text { - Section IV - SR C - 16:20-16:40 }
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## Few hamiltonian cycles in $(k, l)$-regular graphs

Jorik Jooken (KU Leuven)

Sheehan conjectured that every hamiltonian 4-regular graph contains at least two hamiltonian cycles, whereas Fleischner showed the existence of hamiltonian graphs in which every vertex has degree 4 or 14 and in which there is exactly one hamiltonian cycle. This motivates the study of $h_{n}(k, l)$ : the minimum number of pairwise distinct hamiltonian cycles in hamiltonian graphs on $n$ vertices for which every vertex has degree $k$ or $l$. In this work, we disprove a conjecture of Haythorpe about upper bounds for $h_{n}(k, k)$. We also answer a question of Haxell, Seamone and Verstraete and a question of Thomassen related to a method for proving that $h_{n}(k, k) \geq 2$ for appropriate $k$ and $n$. Finally, we present results concerning $h_{n}(k, l)$ for $k \leq 3$. The results from this work were obtained by using theoretical arguments as well as an exhaustive computer search. This is joint work with Jan Goedgebeur, On-Hei Solomon Lo, Ben Seamone and Carol T. Zamfirescu.

# Friday, 13 Oct. 2023 - Time: 16:45-17:05 

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29 \text { - Section I - HS - 16:45-17:05 }
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## Resilience for loose Hamilton cycles

RICHARD LANG (Universität Hamburg)

We study the emergence of loose Hamilton cycles in subgraphs of random hypergraphs. Our main result states that the minimum $d$-degree threshold for loose Hamiltonicity relative to the random $k$ uniform hypergraph $G_{n, p}^{(k)}$ coincides with its dense analogue whenever $p \geq n^{-(k-1) / 2+\gamma}$ for some $\gamma>0$. The value of $p$ is approximately tight for $d>(k+1) / 2$. This is particularly interesting because the dense threshold itself is not known beyond the cases when $k=3$ or $d \geq k-1$.

Joint work with José D. Alvarado, Yoshiharu Kohayakawa, Guilherme Oliveira Mota and Henrique Stagni.
$30-$ Section II - SR A - 16:45-17:05

## On existence of extended perfect codes

Konstantin Vorob'ev (Institute of Mathematics and Informatics, Bulgaria)

A code $C$ in the Hamming graph $H(n, q)$ is an extended 1-perfect code if its projection on each coordinate position (equivalently, a puncturing) equals an 1-perfect code. It follows from [3] and [4] that if $q$ is prime power, then a code in $H(n, q)$ is 1-perfect if and only if it has parameters of the form $\left(n=\frac{q^{r}-1}{q-1}, q^{n-r}, 3\right)_{q}$, for some nonnegative integer $r$. Therefore, possible extended 1-perfect codes for such $q$ have parameters $\left(\frac{q^{r}-1}{q-1}+1, q^{n-r}, 3\right)_{q}$. The parameters of known families of extended codes are the following (see e.g. [2]):

1. $r=1, q \geq 2$ (one vertex code);
2. $r=2, q=2^{m}, m \geq 1$;
3. $r \geq 1, q=2$ (binary extended 1-perfect codes).

Recently, Bespalov in [1] proved the nonexistence of 1-perfect codes for $q=3,4$ and $n>q+2$ (i.e., $r>2$ ) by using techniques from the theory of equitable partitions and their weight distributions. In this work, we develop these ideas and solve the classification problem of admissible parameters of extended 1-perfect codes, when $q$ is prime power. The main result is that the parameters of such codes coincide with those from the list above.

The author was supported by the NSP P. Beron project CP-MACT.

## References:

[1] E. Bespalov, On the non-existence of extended 1-perfect codes and MDS codes, J. Combin. Theory Ser. A, vol. 189 (2022), 105607.
[2] F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes, Amsterdam, Netherlands: North Holland, 1977.
[3] A. Tietäväinen, On the nonexistence of perfect codes over finite fields, SIAM J. Appl. Math., vol. 24, no. 1, pp. 88-96, 1973.
[4] V. Zinoviev and V. Leontiev, The nonexistence of perfect codes over Galois fields, Probl. Control Inf. Theory, vol. 2, no. 2, pp. 123-132, 16-24[Engl. transl.], 1973.

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31-\text { Section III - SR B - 16:45-17:05 }
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## Hyperplane sweeps in Euclidean oriented matroids

Winfried Hochstättler (FernUniversität in Hagen)

In the generalization of linear programming to oriented matroids the simplex algorithm may run into non-degenerate cycles. It is quite obvious that this prevents the possibility of a topological sweep of the vertices of the feasible region. An oriented matroid program without such non-degenerate directed cycles is called Euclidean. It is claimed in the literature, and has recently been made precise by us, that this allows for a topological sweep.

In a Euclidean oriented matroid program the improving directions define an acyclic digraph. In this talk we show that any topological sorting (linear extension) of this digraph can be realized by a topological sweep by parallel hyperplanes.
Joint work with Michael Wilhelmi.

## 32 - Section IV - SR C - 16:45-17:05

## Resistance, oddness and colouring defect of snarks

Imran Allie (University of Cape Town)

Let $G$ be a bridgeless cubic graph. The resistance of $G$, denoted $r(G)$, is the minimum number of edges which can be removed from $G$ in order to render 3-edge-colourability. The oddness of $G$, denoted $\omega(G)$, is the minimum number of odd components in a 2 -factor of $G$. The colouring defect of $G$ (or simply, the defect of $G$ ), denoted $\mu_{3}(G)$, is the minimum number of edges not contained in any set of three perfect matchings of $G$. These three parameters are regarded as measurements of uncolourability of snarks, partly because $r(G)=\omega(G)=\mu_{3}(G)=0$ if and only if $G$ is 3-edgecolourable. It is also known that $r(G) \geq \omega(G)$ and that $\mu_{3}(G) \geq \frac{3}{2} \omega(G)$ [Fiol et al (2018), Jin and Steffen (2017)]. We have shown that the ratio of oddness to resistance can be arbitrarily large for non-trivial snarks [Allie (2022)]. It has also been shown that the ratio of the defect to oddness can be arbitrarily large for non-trivial snarks, although this result was only shown for graphs with oddness equal to 2 [Karabas et al (2021)]. In the same paper, the question was posed whether there exists non-trivial snarks for given resistance $r$ or given oddness $\omega$, and arbitrarily large defect. In this talk, we present an outline of a proof of a stronger result: For any positive integers $r \geq 2$, even $\omega \geq r$, and $d \geq \frac{3}{2} \omega$, there exists a non-trivial snark $G$ with $r(G)=r, \omega(G)=\omega$ and $\mu_{3}(G) \geq d$.

## Saturday, 14 Oct. 2023 - Time: 10:30-10:50

# The Iteration Number of the Weisfeiler-Leman Algorithm 

Moritz Lichter (RWTH Aachen University)

The $k$-dimensional Weisfeiler-Leman ( $k$-WL) algorithm is a simple combinatorial algorithm that was originally designed as a graph isomorphism heuristic. It naturally finds applications in Babai's quasipolynomial-time graph isomorphism algorithm, practical graph isomorphism solvers, and algebraic graph theory. However, it also has surprising connections to other areas such as logic, proof complexity, combinatorial optimization, and machine learning.

The algorithm iteratively computes a coloring of the $k$-tuples of vertices of a graph. Of particular interest is the number of iterations required by the algorithm. Since Fürer's linear lower bound from 2001, it has been an open question whether there is a super-linear lower bound for the iteration number for $k$-WL on graphs. After providing an overview of existing bounds in this talk, I answer this question and present a new $\Omega\left(n^{k / 2}\right)$ lower bound for the iteration number of $k$-WL on $n$-vertex graphs for every $k$. The talk is based on joint work with Martin Grohe, Daniel Neuen, and Pascal Schweitzer.

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34 \text { - Section II - SR B - 10:30-10:50 }
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## The Lovász-Cherkassky theorem in infinite graphs

## Attila Joó (Universität Hamburg)

Lovász and Cherkassky discovered independently that if $G$ is a finite graph and $T \subseteq V(G)$ such that there are no vertices of odd degree out of $T$, then the maximal number of edge-disjoint paths connecting distinct vertices in $T$ is $\frac{1}{2} \sum_{t \in T} \lambda(t, T-t)$ where $\lambda$ is the local edge-connectivity function. We generalize their theorem to infinite graphs in a structural way in the spirit of the infinite version of Menger's theorem by Aharoni and Berger.

## On $d$-dimensional nowhere-zero $r$-flows on a graph

Davide Mattiolo (KU Leuven)

A $d$-dimensional nowhere-zero $r$-flow, or $(r, d)$-NZF, on a graph $G$ is a flow where the value on each edge is an element of $\mathbb{R}^{d}$ whose (Euclidean) norm lies in the interval $[1, r-1]$. Such a notion is a natural generalization of the well-known concept of circular nowhere-zero $r$-flow (i.e. $d=1$ ). The $d$-dimensional flow number $\phi_{d}(G)$ of a bridgeless graph $G$ is the minimum of the real numbers $r$ such that $G$ admits an $(r, d)$-NZF. For every bridgeless graph $G$, the 5 -flow Conjecture claims that $\phi_{1}(G) \leq 5$, while a conjecture by Jain suggests that $\phi_{d}(G)=1$, for all $d \geq 3$. In this talk we address the problem of finding a possible upper-bound also for the remaining case $d=2$. This is a joint work with Giuseppe Mazzuoccolo, Gloria Tabarelli and Jozef Rajník.

## Saturday, 14 Oct. 2023 - Time: 10:55-11:15

# Efficiently finding low-sum copies of spanning forests in zero-sum complete graphs via conditional expectation 

Dieter Rautenbach (Ulm University)

For a fixed positive $\epsilon$, we show the existence of a constant $C_{\epsilon}$ with the following property: Given a $\pm 1$-edge-labeling $c: E\left(K_{n}\right) \rightarrow\{-1,1\}$ of the complete graph $K_{n}$ with $c\left(E\left(K_{n}\right)\right)=0$, and a spanning forest $F$ of $K_{n}$ of maximum degree $\Delta$, one can determine in polynomial time an isomorphic copy $F^{\prime}$ of $F$ in $K_{n}$ with $\left|c\left(E\left(F^{\prime}\right)\right)\right| \leq\left(\frac{3}{4}+\epsilon\right) \Delta+C_{\epsilon}$.
The presented results are joint work with Johannes Pardey.

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37 \text { - Section II - SR B - 10:55-11:15 }
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## Improved bounds for Nash-Williams's orientation conjecture in infinite graphs

MAx PitZ (Universität Hamburg)

Nash-Williams proved in 1960 that a finite graph admits a $k$-arc-connected orientation if and only if it is $2 k$-edge-connected. He conjectured that the same result should hold for all infinite graphs, too, but no progress had been made until 2016, when C. Thomassen proved that all $8 k$-edge-connected infinite graphs admit a $k$-arc connected orientation. In this talk, I show how to establish the optimal bound $2 k$ in Nash-Williams's conjecture for the graph class of all locally finite graphs with countably many ends. The main ingredients are a close analysis of liftings and orientations in finite graphs. This is joint work with A. Assem (Waterloo) and M. Koloschin (Hamburg).

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38-\text { Section III - SR C - 10:55-11:15 }
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## On the Frank Number and Nowhere-Zero Flows on Graphs

Jarne Renders (KU Leuven Kulak)

An edge $e$ of a graph $G$ is called deletable for some orientation $o$ if the restriction of $o$ to $G-e$ is a strong orientation. Inspired by an open problem of Frank, in 2021 Hörsch and Szigeti proposed a new parameter for 3-edge-connected graphs, called the Frank number, which refines $k$-edge-connectivity. The Frank number is defined as the minimum number of orientations of $G$ for which every edge of $G$ is deletable in at least one of them.

They showed that every 3 -edge-connected graph has Frank number at most 7 and that in case these graphs are also 3 -edge-colourable the parameter is at most 3 . Here we strengthen both results by showing that every 3 -edge-connected graph has Frank number at most 4 and that every graph which is 3-edge-connected and 3-edge-colourable graph has Frank number 2. The latter also confirms a conjecture by Barát and Blázsik.

Furthermore, we prove two sufficient conditions for cubic graphs to have Frank number 2 and use them in an algorithm to computationally show that the Petersen graph is the only cyclically 4-edgeconnected cubic graph up to 36 vertices having Frank number greater than 2.


[^0]:    11 - Section III - SR B - 11:20-11:40

