

Some explicit and asymptotic formulas related to the generalized arithmetic triangles

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This talk first introduces the m -arithmetic triangle — a generalization of Pascal's triangle, and then presents an explicit formula for the numbers of the m -arithmetic triangle; we have found some asymptotic expansions for these numbers. With a new formula for inversion of analytic functions, which we introduced recently, we get inversion formulas, in which new special numbers \mathcal{A}_n appear. We present a recurrence formula for \mathcal{A}_n using an analogue of the Pascal's triangle, and give a relation formula of \mathcal{A}_n with Bernoulli numbers. We also derive a new explicit formula for Bernoulli numbers. In conclusion, we present the power series expansions for some elementary functions as well as asymptotic expansions of certain special functions that involve \mathcal{A}_n .

Packing densities of layered permutations and the minimum number of monotone sequences in layered permutations

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In this talk, we present two new results of layered permutation densities. The first one generalizes theorems from Hästö and Warren to compute the permutation packing of permutations with layer sequence $(1^a, l_1, l_2, \dots, l_k)$ such that $2^a - a - 1 \geq k$ (and similar permutations).

The second result is related to the minimum density of monotone sequences of length $k + 1$ in an arbitrarily large permutation is asymptotically $1/k^k$ if we require the largest permutation to be layered. This value is compatible with a Conjecture posed by Myers that estimates the minimum number of monotone sequences of different lengths.

Perfect f -Matchings and f -Factors in Hypergraphs

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Deciding whether a hypergraph has a perfect f -matching or an f -factor is in general NP-complete. Therefore, we concentrate on two classes of hypergraphs generalizing bipartite graphs.

First, we focus on the perfect f -matching problem in mengerian hypergraphs. We give a necessary and a sufficient condition for the existence of a perfect f -matching which are tight for uniform mengerian hypergraphs.

The second class of hypergraphs we investigate are balanced hypergraphs. Conforti, Cornuéjols, Kapoor, and Vušković generalized Hall's Theorem for the existence of a perfect matching in bipartite graphs to balanced hypergraphs. We show how this result can be extended to the case of perfect f -matchings and f -factors in uniform balanced hypergraphs. At the end, we compare our results to the characterization of perfect f -matchings and f -factors in bipartite graphs.

Matching-covered graphs and the Robust Matching Problem

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We consider the perfect matching problem on a graph under uncertainty, where uncertainty is given by a collection of subsets of edges. Each subset from the collection defines a scenario that, if emerged, leads to a deletion of the corresponding edges from the underlying graph. An edge subset from the graph is called a robust perfect matching if it contains a perfect matching for each scenario. Our goal is to determine a robust perfect matching of minimum cardinality.

In this talk, we discuss properties for feasible and optimal solutions, their connection to matching-covered graphs and present complexity results. We focus on bipartite graphs and on structured sets of scenarios.

Linear Polyomino Achievement

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For a given set $P = \{p_1, \dots, p_n\}$ of integers the following achievement game will be considered. Two players A (first move) and B alternately color the integers. Player A wins if he achieves a copy of P (that is $\{p_1 + k, \dots, p_n + k\}$ or $\{k - p_n, \dots, k - p_1\}$ for an integer k) in his color and B wins otherwise. The polyomino P is called a winner if there exists a winning strategy for A . Otherwise there exists a strategy for B to prevent A from winning and then P is called a loser.

Joint work with Christian Löwenstein, Dirk Meierling, and Robert Scheidweiler.

Efficient Domination for P_6 -Free Graphs – the Final Result

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(joint work with RAFFAELE MOSCA, University of Pescara, Italy)

The NP-complete EXACT COVER problem for a hypergraph $H = (V, \mathcal{E})$ asks for the existence of a subset $\mathcal{F} \subseteq \mathcal{E}$ of hyperedges covering every vertex of V exactly once. The NP-complete EFFICIENT DOMINATION (ED) problem for a graph $G = (V, E)$ corresponds to the Exact Cover problem for the closed neighborhood hypergraph of G . It is well known that the ED problem is NP-complete for claw-free graphs, for bipartite graphs as well as for chordal graphs.

Obviously, if a graph F is claw-free and cycle-free then F is the disjoint union of paths (called a *linear forest*). Thus, ED is NP-complete for F -free graphs whenever F is not a linear forest. From a standard reduction, it follows that ED is NP-complete for $2P_3$ -free chordal graphs and thus also for P_7 -free chordal graphs. For P_6 -free graphs, the complexity of ED was the last open question; for all other linear forests F , ED is either NP-complete or solvable in polynomial time.

Recently, Lokshantov, Pilipczuk, and van Leeuwen, using their sub-exponential algorithm for Maximum Weight Independent Set on P_6 -free graphs, have shown that weighted ED can be solved in polynomial time for P_6 -free graphs (their time bound is something like $O(n^{500})$). Independently, Raffaele Mosca and I, using a direct approach, have shown that weighted ED can be solved in time $O(n^6 m)$. Our approach first reduces the ED problem on P_6 -free graphs to the one on P_6 -free unipolar graphs (a *unipolar graph* has a partition into a clique and the disjoint union of cliques) and in a second step solves ED on P_6 -free unipolar graphs in time $O(n^4 m)$.

This finally leads to a dichotomy for the complexity of ED on F -free graphs.

Colouring graphs with constraints on local connectivity

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We consider several generalisations of the class of graphs with maximum degree at most k — each defined by constraining the local connectivity of some, or all, pairs of vertices — and the complexity of k -colouring for a graph in one of these classes. A graph has *maximal local connectivity* k (respectively, *maximal local edge-connectivity* k) if no pair of distinct vertices have more than k internally disjoint (respectively, edge-disjoint) paths between them. We characterise the 3-colourable graphs with maximal local edge-connectivity 3, the 3-colourable 3-connected graphs with maximal local connectivity 3, and the k -colourable k -connected graphs with maximal local edge-connectivity k . It follows that there is a polynomial-time algorithm that, for a graph in one of these classes, finds a 3- or k - colouring, or determines that none exists. On the other hand, deciding the k -colourability of minimally k -connected graphs, or deciding the 3-colourability of $(k - 1)$ -connected graphs with maximal local connectivity k , is shown to be NP-complete.

This is joint work with Pierre Aboulker, Frédéric Havet, Dániel Marx, and Nicolas Trotignon.

On mark sequences in multidigraphs

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An r -digraph D is an orientation of a multigraph that is without loops and contains at most r edges between any pair of distinct vertices. The mark of a vertex v_i in D is $p_i = r(n - 1) + d_i^+ - d_i^-$, where d_i^+ and d_i^- are respectively the outdegree and indegree of v_i . The sequence of marks is called the mark sequence of D . One representation of a multidigraph is a competition in which n participants play each other r times and the result includes the ties. We characterize mark sequences in digraphs, which also result in construction algorithms.

Forbidding substructures for χ -boundedness

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The clique number is a trivial lower bound for the chromatic number of a graph. Since Erdős showed the existence of graphs with arbitrarily high chromatic number and arbitrarily high girth (so clique number is 2), in general, the chromatic number of a graph cannot be upper bounded by a function of its clique number. A class of graphs is said to be χ -bounded if such a function exists.

Vertex-minor and pivot-minors are graph containment properties such as (induced) subgraphs, subdivisions, and minors. Geelen conjectured that for any fixed graph H , the class of graphs with no H -vertex-minor is χ -bounded. This conjecture was known to be true only for one graph (proved by Dvořák and Král), but recently Chudnovsky, Scott, and Seymour proved it for any cycle. We add another class of graphs for which Geelen's Conjecture is true, namely, fan graphs.

We also ask the following question of whether Geelen's Conjecture can be generalized to pivot-minors: for any fixed graph H , are the class of graphs with no H -pivot-minor χ -bounded? We give some positive evidence to this question by proving that it is true for all cycles, which is a strengthening of the aforementioned result by Chudnovsky, Scott, and Seymour. This result can also be viewed as a partial result of the last open conjecture among the three conjectures made by Gyárfás' in 1985.

Finding Permutation Graph Colorings with a Minimum Number of Unicolored Chains

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For a permutation π that permutes $\{0, \dots, n-1\}$, a chain is an inclusion maximal subset C of the range $\{0, \dots, n-1\}$ of consecutive numbers, such that π^{-1} restricted to C is monotonely increasing. The number of chains of a permutation is interesting in connection to minimize the number of tracks in a hump yard to get cars sorted. For a permutation π and a coloring of the corresponding permutation graph, a unicolored chain is an inclusion maximal subset C of the range of the permutation of consecutive numbers of the same color, such that π^{-1} restricted to C is monotonically increasing. An algorithm is presented to find, given a permutation and a color number at least as large as the chromatic number of the corresponding permutation graph, a coloring of the permutation with as many colors as the color number, such that the number of unicolored chains is minimized. Details will be presented during the talk.

Counting connected dominating sets

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A vertex set W of a graph $G = (V, E)$ is called connected dominating if every vertex in $V \setminus W$ has a neighbour in W and the induced subgraph $G[W]$ is connected. The connected domination polynomial is the ordinary generating function for the number of connected dominating sets of a graph. In this talk we present some basic properties of the connected dominating polynomial. Furthermore, we show some results which characterize the essential and irrelevant vertices and edges of a graph in respect to the calculation of the connected domination polynomial.

Symmetries of Monocoronal Tilings

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The study of tilings using a single tile shape, resp. vertex shape, resp.... has a long history. Whereas a classification of tilings of the Euclidean plane by pairwise congruent tiles is still incomplete, a complete classification of tilings of the Euclidean plane with pairwise congruent vertex corona has been established in 2014 (F.-Garber, Symmetries of Monocoronal Tilings, submitted/accepted, arXiv:1402.4658).

The vertex corona of a vertex in a tiling is the patch of all tiles adjacent to this vertex. A tiling where all vertex coronae are congruent (reflections allowed) is called *monocoronal*. A tiling where all vertex coronae are directly congruent (reflections forbidden) is called *monocoronal wrt rigid motions*. In this talk the classification of monocoronal tilings in the Euclidean plane is presented. Moreover some results on monocoronal tilings in hyperbolic spaces and higher dimensional Euclidean spaces are given.

r -dynamic chromatic number of some line graphs

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An r -dynamic coloring of a graph G is a proper coloring c of the vertices such that $|c(N(v))| \geq \min\{r, \deg(v)\}$, for each $v \in V(G)$. The r -dynamic chromatic number of a graph G is the minimum k such that G admits an r -dynamic coloring with k colors. In this paper, we obtain the r -dynamic chromatic number of line graph of helm graphs H_n for all r between minimum and maximum degree of H_n . Moreover, our proofs are constructive, what means that we give also polynomial time algorithms for the appropriate coloring. Finally, as the first, we define equivalent model for edge coloring.

Finding minimal obstructions to graph colouring

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For several graph classes without long induced paths there exists a finite forbidden subgraph characterisation for k -colourability. Such a finite set of minimal obstructions allows to provide a “no-certificate” which proves that a graph is not k -colourable.

In this talk we will present a new algorithm for generating all minimal forbidden subgraphs for k -colourability for given graph classes.

We will show how the new generation algorithm has been applied to fully characterise the forbidden subgraphs for k -colourability of various classes of graphs without long induced paths. Using this algorithm (combined with new theoretical results) we haven proven amongst others that there are 24 minimally non-3-colourable graphs in the class of P_6 -free graphs, which solves an open problem posed by Golovach et al.

This is joint work with Maria Chudnovsky, Oliver Schaudt and Mingxian Zhong.

Goldberg, Coxeter and a General Approach to Operations on Polyhedra

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Most likely already Euclid used operations on polyhedra to produce new polyhedra from old ones by preserving symmetries. Some well-known operations are *dual*, *ambo*, *truncate*, and other operations as classified by Conway. The *Goldberg/Coxeter operations*, used to construct icosahedral fullerenes from the icosahedron, are examples of more complicated operations. Describing operations as *chamber operations* on the barycentric subdivision of a tiling gives a general definition that makes it possible to answer several interesting questions and allows further research. *E.g.* Which operations can create more symmetry? Can the Goldberg operation be generalized to other regular tilings? Which operations can be decomposed into simpler operations?

Cuts and cycles in transitive graphs

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We discuss the connections between the cycle space and the cut space of transitive graphs. In particular, we will see that the cut space of a transitive graph G is a finitely generated $\text{Aut}(G)$ -module as soon as the same holds for the cycle space.

In addition, we discuss accessibility in transitive locally finite graphs: when does there exist some positive integer n such that any two ends can be separated by removing at most n edges? We use our previously mentioned result to see that this is the case if the cycle space is generated by cycles of bounded length. It turns out that this condition on the cycle space is satisfied by various natural classes of graphs.

The Minimum Crossing Number of the Petersen Graph

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A drawing $D(G)$ of a graph G is a representation of G in the plane if vertices are mapped into distinct points and edges are curves connecting the corresponding points in such a way that two curves have at most one point in common either an endpoint or a point of intersection (crossing). The maximum crossing number $CR(G)$ is the largest number of crossings over all drawings of $D(G)$. Here $CR(P) = 68$ will be proved for the Petersen graph P .

Bicircular Matroids are 3-colorable

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Hugo Hadwiger proved that a graph which is not 3-colorable must have a K_4 -minor and conjectured that a graph which is not k -colorable must have a K_{k+1} -minor. Meanwhile the class of graphs without K_4 -minor has been identified as the class of series-parallel networks.

Together with Nešetřil we extended the theory of colorings and nowhere-zero-flows to oriented matroids.

We generalize the notion of being series-parallel to oriented matroids and show that generalized series-parallel oriented matroids are 3-colorable.

Finally, we prove that every orientation of a bicircular matroid is generalized series-parallel.

Rainbow connection and forbidden subgraphs

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A connected edge-coloured graph G is rainbow-connected if any two distinct vertices of G are connected by a path whose edges have pairwise distinct colours. The rainbow connection number $rc(G)$ of G is the minimum number of colours such that G is rainbow-connected. We consider families \mathcal{F} of connected graphs for which there is a constant $k_{\mathcal{F}}$ such that, for every connected \mathcal{F} -free graph G , $rc(G) \leq \text{diam}(G) + k_{\mathcal{F}}$, where $\text{diam}(G)$ is the diameter of G . In this talk we give a complete answer for any finite family \mathcal{F} .

Unions of 1-factors in r -graphs

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The generalized Berge-Fulkerson conjecture states that every r -graph has $2r$ 1-factors such that each edge is contained in precisely two of them. This conjecture is shown to be equivalent to the statement that every r -graph can be covered by $2r - 1$ 1-factors. In this talk, we obtain, for any positive integers $r \geq 3$ and k , a lower bound of the fraction of edges covered by k 1-factors in r -graphs. Moreover, it was announced by Kaiser, Král and Norine [Unions of perfect matching in cubic graphs, Topics in Discrete Mathematics, in: Algorithms Combin., vol. 26, Springer, Berlin, 2006, pp. 225 - 230] and completely proved by Mazzuocolo [Covering a cubic graph with perfect matchings, Discrete Mathematics 313 (2013) 2292 - 2296] a lower bound for the fraction of edges covered by k 1-factors in bridgeless cubic graphs (i.e., 3-graphs). Our result extends this to r -graphs with $r \geq 3$.

The Erdős-Pósa property of rooted minors

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A family \mathcal{H} of graphs is said to have the *Erdős-Pósa property* if there is a function $f : \mathbb{N} \rightarrow \mathbb{N}$ so that any graph contains k disjoint subgraphs that are isomorphic to graphs in \mathcal{H} , or if it contains a vertex set of size at most $f(k)$ meeting all such subgraphs. In their famous graph minors series Robertson and Seymour prove that the family of all H -expansions (all graphs that contain H as a minor) has the Erdős-Pósa property if and only if H is planar. We extend this result to rooted H -expansions and describe exactly for which rooted graphs H the family of rooted H -expansions has the Erdős-Pósa property.

This is joint work with Henning Bruhn and Oliver Schaudt.

Choosability in Signed Planar Graph

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We study the choosability of signed planar graphs. We prove that every signed planar graph is 5-choosable and that there is a signed planar graph which is not 4-choosable while the unsigned graph is 4-choosable. For each $k \in \{3, 4, 5, 6\}$, every signed planar graph without circuits of length k is 4-choosable. Furthermore, every signed planar graph without circuits of length 3 and of length 4 is 3-choosable. We construct a signed planar graph with girth 4 which is not 3-choosable but the unsigned graph is 3-choosable.

These results are joint work with Ligang Jin and Eckhard Steffen.

Drawings of planar graphs with prescribed face areas

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Let G be a planar graph with a planar drawing. The drawing of G yields a set of inner faces F' . Let $A: F' \rightarrow \mathbb{R}^+$ be a face-area assignment. We are interested in (straight-line) drawings of G such that every face $f \in F'$ has area $A(f)$.

If there exists a realizing drawing of G for A with $A(f) := 1 \forall f \in F'$, G is *equiareal*. G is *area-universal* if for every area-assignment A there exists a realizing straight-line drawing of G .

It is known that not all plane graphs are area-universal. In particular, not all plane graphs are equiareal. Hence, we investigate relaxations. One possibility is to allow the edges to bend, i.e., an edge is the concatenation of several segments. How many bends are sufficient?

We show that it is enough to allow one bend per edge in order to realize any face-area assignment of every plane graph.

Linear structure of graphs and the knotting graph

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Many important graph classes, as interval graphs, comparability graphs and AT-free graphs, show some kind of linear structure. In this talk we try to capture the notion of linearity and show some algorithmic implications.

In the first part of the talk we discuss the notion of linearity of graphs and give some motivation for its usefulness for particular graph classes. Then we consider the knotting graph, a combinatorial structure that was defined by Gallai long ago and that has various nice properties with respect to our notion of linearity. Next we define intervals of graphs, a concept that generalizes betweenness in graphs—a crucial notion for capturing linear structure in graphs. In the last part we give a practical example of how to use linear structure of graphs algorithmically. In particular we show how to use these structural insights for finding maximum independent sets in AT-free graphs in $O(nm')$ time, where m' denotes the number of non-edges of the graph G .

Biconnected Reliability - An Alternative Approach to Count Biconnected Graphs

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Biconnected reliability is the probability that a given probabilistic graph is biconnected. If all edges fail independently with the same probability, we obtain the biconnected reliability polynomial, whose coefficients count the number of biconnected spanning subgraphs with a given number of edges. In general, the calculation of this polynomial results in the enumeration of the complete state space. A method will be presented which enables a more efficient calculation for certain graph classes by exploiting their highly symmetric structure. Applied on complete graphs and complete bipartite graphs, this method will be used to count the number of biconnected and biconnected bipartite graphs with n vertices.

Order of first occurrence and minors of functions

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The idea of listing the elements appearing in a sequence of data in the order of first occurrence is employed in many situations in science, humanities, and everyday life. Denoting by A^* the set of all string over a set A and by A^\sharp the subset of A^* comprising the strings without repeated letters, this idea is formalized by the function $\text{of}_0: A^* \rightarrow A^\sharp$ that maps each string $a_1a_2 \dots a_n \in A^*$ to the string obtained from $a_1a_2 \dots a_n$ by removing repeated occurrences of letters, keeping only the first occurrence of each letter. A function $f: A^n \rightarrow B$ is *determined by the order of first occurrence* if it is decomposable as $f = f^* \circ \text{of}_0|_{A^n}$ for some map $f^*: A^\sharp \rightarrow B$.

This talk reports some recent work in which the order of first occurrence and the function of_0 have come up. In particular, we discuss identification minors of functions of several arguments and a related reconstruction problem. A remarkable property of functions determined by the order of first occurrence is that they have a unique identification minor. The function of_0 also serves as an interesting example of an associative string function.

Star packings and $(2,2)$ domination in graphs; new upper bounds

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For $k \geq 1$, a k -star is the complete bipartite graph $K_{1,k}$ of order $k + 1$. For a graph G of minimum degree at least k , a k^+ -star packing of G , is a collection of vertex-disjoint stars of G all of order at least $k + 1$. A subset D of the vertex set of a graph G is a $(2, 2)$ -dominating set if every vertex of $V(G) \setminus D$ is at distance at most 2 from at least 2 vertices of D . The $(2, 2)$ -domination number $\gamma_{2,2}(G)$ of G is the minimum cardinality of a $(2, 2)$ -dominating set of G . In this paper, by using some k^+ -star packings, we give new upper bounds on the $(2, 2)$ -domination number of a graph of minimum degree $\delta \geq 4$.

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Sum list colorings of products of graphs

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Let $G = (V, E)$ be a simple graph and for every vertex $v \in V$ let $L(v)$ be a list of available colors. G is called *L-colorable* if there is a proper vertex coloring c with $c(v) \in L(v)$ for all $v \in V$. A function $f : V \rightarrow \mathbb{N}$ is called a *choice function* of G if G is L -colorable for every list assignment L with $|L(v)| = f(v)$ for all $v \in V$. Set $\text{size}(f) = \sum_{v \in V} f(v)$ and define the *sum choice number* $\chi_{sc}(G)$ as the minimum of $\text{size}(f)$ over all choice functions f of G .

A general upper bound for $\chi_{sc}(G)$ of a graph G is $\chi_{sc}(G) \leq |V| + |E|$. A graph G is called *sc-greedy* if $\chi_{sc}(G) = |V| + |E|$.

In this talk we consider the Cartesian product, the direct product, the strong product, and the lexicographic product of two graphs, and we ask which of them are *sc-greedy* graphs.

This is joint work with Arnfried Kemnitz and Margit Voigt.

Bounded diameter arboricity

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(joint work with Luke Postle)

The arboricity of a graph G is the smallest number of forests needed to cover the edges of G . We introduce a new concept called the *diameter- d arboricity* of G , which is the smallest number of forests needed to cover the edges of G under the additional constraint that the diameter of the trees is at most d . If d is greater than the size of G , then the diameter- d arboricity of G is just the usual arboricity. For $d = 2$, the diameter- d arboricity of a graph is the same as the well-studied star arboricity.

We conjecture that for every natural number k there exists a number $f(k)$ such that every graph with arboricity k has diameter- $f(k)$ arboricity at most $k + 1$. We verify this conjecture for $k \leq 3$ by giving an algorithm that shows $f(3) \leq 22$. As a corollary we get that every planar graph has diameter-22 arboricity at most 4, i. e. every planar graph can be decomposed into 4 forests in which each tree has diameter at most 22.

Improved Approximation Algorithm for Minimum Feedback Vertex Sets in Tournaments

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We consider the minimum feedback vertex set problem in tournaments, which finds applications in ranking scenarios. This problem is NP-hard to solve exactly, and Unique Games-hard to approximate by a factor better than two. We present an approximation algorithm for this problem that improves on the previously best known ratio $5/2$, given by Cai et al. in FOCS 1998 / SICOMP 2001.

Bounds for generalized nonrepetitive sequences

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A sequence that does not contain a repetition of any length is called *nonrepetitive* or *Thue*. In this talk we consider a generalization of such sequences. A sequence S is called *k-Thue* if every subsequence of S , in which two consecutive terms are at indices of common differences from the set $\{1, 2, \dots, k\}$ in S , is also Thue.

It was conjectured that $k + 2$ symbols are enough to construct an arbitrarily long *k-Thue* sequence and proved that the conjecture is true for $k = 1, 2, 3$ and 5 . We present a construction of *4-Thue* sequences on 6 symbols, which confirms this conjecture also for $k = 4$. Moreover, we discuss known upper bounds on the number of symbols that suffice to construct such sequences.

This is a joint work with Jaka Kranjc, Borut Lužar and Roman Soták.

On an anti-Ramsey threshold for sparse graphs with one triangle

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For graphs G and H , let $G \xrightarrow[p]{rb} H$ denote the property that for every *proper* edge-colouring of G (with an arbitrary number of colours) there is a *rainbow* copy of H in G , that is, a copy of H with no two edges of the same colour. In 2014 the authors showed that, for every graph H , the threshold function $p_H^{rb} = p_H^{rb}(n)$ of this property for the binomial random graph $G(n, p)$ is asymptotically at most $n^{-1/m^{(2)}(H)}$. Recently, it was proved by Nenadov, Person, Škorić and Steger that if H is a cycle with at least 7 vertices or a complete graph with at least 19 vertices, then $p_H^{rb} = n^{-1/m^{(2)}(H)}$. Here we prove that there exists a fairly rich, infinite family of graphs F containing a triangle such that $p_F^{rb} \ll n^{-1/m^{(2)}(F)}$.

Joint work with Y. Kohayakawa and P. B. Konstantinidis.

Universality in random and sparse hypergraphs

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Finding spanning subgraphs is a well studied problem in random graph theory, in the case of hypergraphs less is known and it is natural to study the corresponding spanning problems for random hypergraphs. We study universality, i.e. when does an r -uniform hypergraph contain *any* hypergraph on n vertices and with maximum vertex degree bounded by Δ ?

For $\mathcal{H}^{(r)}(n, p)$ we show that this holds for $p = \omega((\ln n/n)^{1/\Delta})$ a.a.s. Furthermore we derive from explicit constructions of universal graphs due to Alon, Capalbo constructions of universal hypergraphs of size almost matching the lower bound $\Omega(n^{r-r/\Delta})$.

This is joint work with Samuel Hetterich and Yury Person.

Minimum H -decompositions of graphs

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Let $\phi_H(G)$ be the minimum number of parts that are needed to partition the edge set of a graph G such that every part is either a single edge or is isomorphic to H . The problem is what graph G on n vertices maximizes $\phi_H(G)$? This problem is tightly related to the classical extremal function $\text{ex}(n, H)$. I review its history and the methods employed in its study and I will also talk about its rainbow variants.

Joint work with Lale Özkahya.

On Laplacian energy of graphs

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For a simple graph $G(V, E)$ with n vertices and m edges, the adjacency matrix $A = (a_{ij})$ of G is a $(0, 1)$ -square matrix of order n whose (i, j) -entry is equal to 1, if vertex v_i is adjacent to vertex v_j and equal to 0, otherwise. If $D(G) = \text{diag}(d_1, d_2, \dots, d_n)$ is the diagonal matrix associated to G , where $d_i = \deg(v_i)$, the matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix and its spectrum is called the Laplacian spectrum of the graph G . The Laplacian energy of a graph G as put forward by Gutman and Zhou is defined as $LE(G) = \sum_{i=1}^n |\mu_i - \frac{2m}{n}|$. The motivation for Laplacian energy comes from graph energy. This quantity, which is an extension of graph-energy concept, has found remarkable chemical applications beyond the molecular orbital theory of conjugated molecules. In this talk, we discuss several bounds for Laplacian energy including the recent stronger bounds obtained by the author.

Graphs in which some or every maximum matching is uniquely restricted

DIETER RAUTENBACH

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A matching M in a graph G is uniquely restricted if there is no matching M' in G that is distinct from M but covers the same vertices as M . Solving a problem posed by Golubic, Hirst, and Lewenstein, we characterize the graphs in which some maximum matching is uniquely restricted. Solving a problem posed by Levit and Mandrescu, we characterize the graphs in which every maximum matching is uniquely restricted. Both our characterizations lead to efficient recognition algorithms for the corresponding graphs.

The presented results are joint work with Lucia D. Penso (Ulm University) and Uéverton dos Santos Souza (UFF, Niteroi, Brazil).

A new proof of Seymour's 6-flow theorem

EDITA ROLLOVÁ

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A nowhere-zero k -flow on a graph G is an assignment of a direction of edges of G and a valuation of the directed edges by integers from $\{\pm 1, \dots, \pm(k-1)\}$ in such a way that in every vertex of G the sum of incoming values equals the sum of outgoing ones. It is conjectured by Tutte that every bridgeless graph admits a nowhere-zero 5-flow. The best general approach to this conjecture is a result of Seymour stating that every bridgeless graph admits a nowhere-zero 6-flow. In this talk we provide a different proof of Seymour's result.

Joint work with Matt DeVos and Robert Šámal.

Obstructions for three-coloring graphs with one forbidden induced subgraph

OLIVER SCHAUDT

Universität zu Köln

The complexity of coloring graphs without long induced paths is a notorious problem in algorithmic graph theory, an especially intriguing case being that of 3-colorability. So far, not much was known about certification in this context.

We prove that there are only finitely many 4-critical P_6 -free graphs, and give the complete list that consists of 24 graphs. In particular, we obtain a certifying algorithm for 3-coloring P_6 -free graphs, which solves an open problem posed by Golovach et al. Here, P_6 denotes the induced path on six vertices.

Our result leads to the following dichotomy theorem: if H is a connected graph, then there are finitely many 4-critical H -free graphs if and only if H is a subgraph of P_6 . This answers a question of Seymour.

The proof of our main result involves two distinct automatic proofs, and an extensive structural analysis by hand.

This is joint work with Maria Chudnovsky, Jan Goedgebeur and Mingxian Zhong.

Chromatic number of P_5 -free graphs: χ -binding functions

INGO SCHIERMEYER

Technische Universität Bergakademie Freiberg

In this talk we study the chromatic number of P_5 -free graphs. Gyárfas has shown the following

Theorem Let G be a P_k -free graph for $k \geq 4$ with clique number $\omega(G) \geq 2$. Then $\chi(G) \leq (k - 1)^{\omega(G)-1}$.

We will show that there is a polynomial χ -binding function for several subclasses of P_5 -free graphs. Our main result is the following.

Theorem Let G be a P_5 -free graph of order n and clique number $\omega(G)$. If G is

(i) *Claw*-free or (ii) *Paw*-free or (iii) *Diamond*-free or (iv) *Dart*-free or

(v) *Cricket*-free or (vi) Gem^+ -free,

then $\chi(G) \leq \omega^2(G)$.

Here Gem^+ denotes the graph $(K_1 + (K_1 \cup P_4))$.

Minimum $(k - 2)$ -degree conditions for Hamiltonian ℓ -cycles in k -uniform hypergraphs

JACOB SCHNITZER

Universität Hamburg

We study minimum $(k - 2)$ -degree conditions for k -uniform hypergraphs which ensure the existence of Hamiltonian ℓ -cycles, where $\ell < k/2$. We show that every k -uniform hypergraph \mathcal{H} on n vertices with $\delta_{k-2}(\mathcal{H}) \geq \left(\frac{4(k - \ell) - 1}{4(k - \ell)^2} + o(1) \right) \binom{n}{2}$ contains a Hamiltonian ℓ -cycle if $k - \ell$ divides n . This bound is asymptotically best possible.

Circular flow number of signed graphs

MICHAEL SCHUBERT

Universität Paderborn

In the first part of the talk, we determine the circular flow numbers of signed regular graphs. Let \mathcal{F}^c be the set of flow numbers of signed graphs, and \mathcal{F}_d^c be the set of the circular flow numbers of signed d -regular graphs. We show that $\mathcal{F}_{2k+1}^c = (\mathcal{F}^c - [2; 2 + \frac{2}{2k-1}]) \cup \{2 + \frac{1}{k}\}$. In the second part we characterize circular flow numbers in terms of orientations. For an orientation f the boundary is given by $\partial f(v) = \delta_f^+(v) - \delta_f^-(v)$. We will relate the problem of finding a nowhere-zero circular r -flow to the problem of finding orientations f_1, \dots, f_n and proper coefficients $\alpha_1, \dots, \alpha_n$ such that $\sum_{i=1}^n \partial f_i \alpha_i = 0$.

Sharp thresholds for monochromatic cycles

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For a given graph F we consider the family of (finite) graphs G with the *Ramsey property* for F , that is the set of such graphs G with the property that every two-colouring of the edges of G yields a monochromatic copy of F . For F being a triangle Friedgut, Rödl, Ruciński, and Tetali (2004) established the sharp threshold for the Ramsey property in random graphs. We obtained a simpler proof of this result which extends to a more general class of graphs F including all cycles. The proof is based on Friedgut's criteria (1999) for sharp thresholds, and on the recently developed *container method* for independent sets in hypergraphs. The proof builds on some recent work of Friedgut et al. who established a similar result for van der Waerden's theorem. This is joint work with Mathias Schacht.

Minimising the Laplacian p -spectral radius

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The Laplacian of a graph $G = (V, E)$ is the matrix $L = D - A$ where A is the adjacency matrix and D is the degree matrix. The Laplacian p -spectral radius is the number

$$\max_{\|x\|_p=1} x^\top Lx$$

which for $p = 2$ yields the largest eigenvalue. Here we consider a weighted Laplacian with non-negative edge weights summing up to $|E|$ and minimise its p -spectral radius by redistributing the edge weights. We give some properties of the optimal value, the corresponding weighted Laplacian and maximising vectors and thereby generalise work of Fiedler on the case $p = 2$. Most results are for bipartite graphs and for the case $p > 2$.

Niche hypergraphs

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(joint work with CHRISTIAN GARSKE and HANNS-MARTIN TEICHERT, Universität zu Lübeck)

If $D = (V, A)$ is a digraph, its *niche hypergraph* $\mathcal{NH}(D) = (V, \mathcal{E})$ has the edge set $\mathcal{E} = \{e \subseteq V \mid |e| \geq 2 \wedge \exists v \in V : e = N_D^-(v) \vee e = N_D^+(v)\}$. Niche hypergraphs generalize the well-known niche graphs and are closely related to competition hypergraphs as well as double competition hypergraphs. We present several properties of niche hypergraphs of acyclic digraphs.

Signed graph coloring

ECKHARD STEFFEN

(JOINT WORK WITH YINGLI KANG)

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We generalize the well studied notion of the circular chromatic number to signed graphs. This implies a new notion of colorings of signed graphs. Some basic facts on circular colorings of signed graphs and on the circular chromatic number are proved, and differences to the results on unsigned graphs are analyzed.

Furthermore, we determine the chromatic spectrum of signed graphs.

On Wiener index of quadratic line graphs

DRAGAN STEVANOVIĆ

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The graph equation $W(L^k(T)) = W(T)$ for a tree T and a positive integer k has raised considerable interest among graph theorists recently. It has no solution for $k = 1$ due to the 1981 result of Buckley, while the case $k \geq 3$ has been fully resolved only recently in a series of papers of Knor, Mačaj, Potočník and Škrekovski. The case $k = 2$, however, is still largely open. Our goal here is to further the study of the equation $W(L^2(T)) = W(T)$ by providing answers to an open problem of Dobrynin and Melnikov from 2004 on the existence of solutions that have arbitrarily large number of arbitrarily long pendant paths, and to a recent conjecture of Knor and Škrekovski that its solutions are homeomorphic to a finite set of homeomorphically irreducible trees.

Rainbow Partitions and Rainbow Colorings

SARA KISCHNICK AND PETER TITTMANN

Hochschule Mittweida

We consider an undirected graph $G = (V, E)$ together with an edge coloring $\phi : E \rightarrow C$ that assigns a *color* from a finite set $C = \{1, \dots, k\}$ to each edge. A *rainbow path* between two vertices u, v in G is a path for which all its edges are differently colored. The edge coloring $\phi : E \rightarrow \{1, \dots, k\}$ is a *rainbow k -coloring* of G if there exists for any two vertices of G a rainbow path connecting them. We investigate the number of $\rho(G, k)$ of rainbow k -colorings, show that $\rho(G, k)$ is a polynomial in k , compute this number for some special graphs, and establish some general lower and upper bounds.

On the strongest form of a theorem of Whitney for hamiltonian cycles in plane triangulations

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We investigate hamiltonian cycles in triangulations. The central part of the talk is the search for the strongest possible form of Whitney's theorem about hamiltonian triangulations in terms of the decomposition tree defined by separating triangles. Jackson and Yu showed that a triangulation is hamiltonian if this decomposition tree has maximum degree 3. We will decide on the existence of non-hamiltonian triangulations with given decomposition trees for all trees except trees with exactly one vertex with degree $k \in \{4, 5\}$ and all other degrees at most 3. For these cases we show that it is sufficient to decide on the existence of non-hamiltonian triangulations with decomposition tree $K_{1,4}$ or $K_{1,5}$, and we give several restrictions on the structure of such non-hamiltonian triangulations. These results were obtained using a combination of computational results and theoretical results, and both will be explained.

On identifying codes in graphs

ANNEGRET WAGLER

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Some newly emerging applications as, e.g., placing detectors for fault detection in networks or motion/fire detection in buildings, can be modeled by special dominating sets in graphs. An identifying code is a dominating set with the additional property that its intersection with the closed neighborhoods yields distinct node sets.

Determining an identifying code of minimum size is in general NP-complete and remains hard for several graph classes where many other problems are easy to solve, including bipartite graphs, split graphs and interval graphs. Hence, a typical line of attack for this problem is to determine minimum identifying codes of special graphs or to provide bounds for their size.

From a graph theoretical point of view, the problem has been actively studied during the last decade. More recently, also polyhedral studies were initiated. In the latter context, it is advantageous to state the problem in terms of transversals of a suitable hypergraph, and to study its combinatorial properties. In my talk, I will illustrate the interplay of the different approaches (rather than presenting an exhaustive list of results achieved in this context).

Sparsity and dimension

VEIT WIECHERT

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We prove that posets of bounded height whose cover graphs belong to a fixed class with bounded expansion have bounded dimension. Bounded expansion, introduced by Nešetřil and Ossona de Mendez as a model for sparsity in graphs, is a property that is naturally satisfied by a wide range of graph classes, from graph structure theory (graphs excluding a minor or a topological minor) to graph drawing (e.g. graphs with constant book thickness). Therefore, our theorem generalizes a number of results including the most recent one for posets of bounded height with cover graphs excluding a fixed graph as a topological minor (Walczak, SODA 2015). We also show that the result is in a sense best possible, as it does not extend to nowhere dense classes; in fact, it already fails for cover graphs with locally bounded treewidth.

This is a joint work with Gwenaël Joret and Piotr Micek.

Horocyclic products of trees – a quasi-isometry problem, its solution, and further ramifications

WOLFGANG WOESS

Universität Graz

This talk is concerned with infinite, connected, locally finite graphs which admit transitive group actions. In 1989/1990 the speaker posed the following question: is there such a graph which is not quasi-isometric with (= “does not look vaguely like”) a Cayley graph of a finitely generated group. A few years later, Diestel and Leader proposed a construction of what was conjectured to be such an example. It is a horocyclic product of two regular trees with different degrees. It was only in 2012+2013 that the complete answer was given in two impressive papers by Eskin, Fisher and Whyte in the *Annals of Mathematics*, along with further related results.

On the other hand, the horocyclic product of two trees with the same degree d is a Cayley graph, of a lamplighter group (wreath product of the infinite cyclic group with a finite group). This has led to a body of work by the speaker plus coauthors concerning random walks and harmonic functions on those graphs.

Further interesting ramifications concern horocyclic products of more than two trees.

The talk will review some of the main features of this development.