Halin Graphs are proper Touching Triangle Graphs

NIEKE AERTS

Diskrete Mathematik, TU Berlin

A Touching Triangle Graph (TTG) is a representation of a graph such that all vertices are triangles and all edges are side contacts. If the drawing is a tiling of a triangle by triangles then it is called a proper TTG. Drawing a graph as a tiling of triangles by triangles has also gotten attention in the classical setting, i.e. which graphs can be drawn such that all edges are straight line segments and all faces including the outer face, are triangles.
It is known that not all bipartite outerplanar graphs have a proper TTG and for the more restricted class of strongly connected outerplanar graphs[1] a necessary and sufficient condition is known. Halin graphs are a subclass of 2-outerplanar graphs. A graph is Halin if it can be constructed from a plane tree without degree 2 vertices by connected the leaves cyclicly.
We use a result in the classical setting to show that Halin Graphs have a TTG.

[1] A biconnected outerplanar graph is strongly connected if the graph of its chords is connected.

A sparse blow-up lemma

PETER ALLEN

London School of Economics, UK

With Böttcher, Hàn, Kohayakawa and Person, we have proved versions of the Blow-up Lemma applicable to relatively dense subgraphs of sparse random or pseudorandom graphs. In this talk I will explain what a Blow-up Lemma is, how it must be modified for the sparse setting (and what exactly that means), and what applications a sparse Blow-up Lemma has.

Some problems on bispanning graphs

DOMINIQUE ANDRES

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A *bispanning graph* is the union of two edge-disjoint trees with edge sets E1 and E2 on the same vertex set. We consider the maker-breaker game, where the edges in E1 are colored purple and those of E2 blue and the maker tries to exchange all colors. The players move alternately. In a move of the maker a blue edge is colored purple. In the next move the breaker has to recolor a different purple edge blue in such a way that the purple and blue edges form spanning trees again. The breaker wins if the maker cannot achieve her goal in a finite number of moves. We show that the breaker wins if the graph contains a K4 and that on every larger wheel the maker wins. Furthermore, there are graphs where a win depends on the initial partition. The game is motivated by connectivity properties of matroid base exchange graphs and an open conjecture of Neil White. We also discuss an algorithm for the recognition of bispanning graphs and the problem of max-min-partition in weighted bispanning graphs.

This is joint work with Winfried Hochstättler, Verena Jochim, and Markus Merkel.

Disjoint compatibility graph of matchings of points in convex position

ANDREI ASINOWSKI

Freie Universität Berlin

Let X_{2k} be a set of $2k$ labeled points in convex position in the plane. We consider geometric non-intersecting straight-line matchings of X_{2k}. Two such matchings, M and M', are *disjoint compatible* if they do not have common edges, and no edge of M crosses an edge of M'. Denote by DCM_k the graph whose vertices correspond to such matchings, and two vertices are adjacent if and only if the corresponding matchings are disjoint compatible. We study connected components of such graphs from the point of view of their structure, size and number. In particular, we show that for each $k \geq 9$, the connected components of DCM_k form exactly three isomorphism classes - namely, there is a certain number of isomorphic "small components", a certain number of isomorphic "middle-size components", and one "big component".

This is a joint work with Oswin Aichholzer (TU Graz) and Tillman Miltzow (FU Berlin).

Achievement games for polyominoes on Catalan tessellations

JENS-P. BODE

Diskrete Mathematik, TU Braunschweig

Catalan tessellations are the duals of the eight Archimedean tessellations of the plane. A polyomino is a simply edge-connected set of polygons of a tessellation, that is, the set and its complement are edge-connected. Two sets of polygons are considered to be the same polyomino if there is a mapping (generated by translations, rotations, and reflections) of the tessellation onto itself which also maps one of the sets of polygons onto the other one.

For a given polyomino P the following achievement game will be considered. Two players A (first move) and B alternatingly color the polygons (cells) of the corresponding tessellation. Player A wins if he achieves a copy of P in his color and B wins otherwise. The polyomino P is called a winner if there exists a winning strategy for A. Otherwise there exists a strategy for B to prevent A from winning and then P is called a loser.

(joint work with Heiko Harborth)

Tree-width and separations of graphs

THOMAS BÖHME

TU Ilmenau, Institut für Mathematik

A well known theorem (Jordan, 1869) states that every n-vertex tree T has a vertex t such that every component of $T - t$ contains at most $2n/3$ vertices. This was generalized to graphs of bounded tree-width as follows (see Flum/Grohe, 2006). Every n-vertex graph G of tree-width at most k contains a set S of at most $k + 1$ vertices such that every component of $G - S$ contains at most $2n/3$ vertices. Tree-width is minor-monotone, and consequently every l-vertex subgraph of a graph of tree-width at most k can be decomposed into components with at most $2l/3$ vertices by deleting at most $k+1$ vertices. Our main objective is to prove a partial converse of this statement. The proof is non-constructive and uses the grid minor theorem (Robertson/Seymour, 1986) stating that for every positive integer m there is a number $g(m)$ such that every graph of tree-width at least $g(m)$ has a minor isomorphic to an $m \times m$-grid. We also discuss similar questions related to the well-known planar separator theorem (Lipton/Tarjan, 1977).

The talk is based on joint work with Jens Schreyer (Ilmenau).

Improved counting relative to pseudorandom graphs

JULIA BÖTTCHER

London School of Economics, UK

Recently, Conlon, Fox and Zhao proved a counting lemma, counting small graphs in ε-regular subgraphs of sparse pseudorandom graphs. This counting lemma has many important applications such as sparse pseudorandom analogues of Turán's Theorem, Ramsey's Theorem and the graph removal lemma.
One key ingredient for the proof of their counting lemma is a regularity inheritance lemma, which states that for most vertices in an ε-regular subgraph of a pseudorandom graph, the neighbourhoods of this vertex form an ε'-regular graph. We improve this regularity inheritance lemma, so that it now applies to graphs with weaker pseudorandomness conditions. This implies an improved counting lemma relative to these pseudorandom graphs.
In the talk I will introduce the concepts of pseudorandomness and ε-regularity, discuss the relevance of counting lemmas, and explain our improvements.

Based on joint work with Peter Allen, Jozef Skokan, Maya Stein.

The Maximum Independent Set Problem in subclasses of subcubic graphs

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The MAXIMUM INDEPENDENT SET problem is known to be NP-hard in general. Many results were published in order to explore the boundary between NP-hard and polynomial time solvable cases. It is well known that the Maximum Independent Set problem is NP-hard in the class of graphs with maximum degree three (also called *subcubic graphs*). In our talk we will present its complexity in hereditary subclasses of subcubic graphs. As a main result we will show that the Maximum Independent Set problem can be solved in polynomial time in several subclasses of subcubic, $S_{2,j,k}$-free graphs, where $S_{i,j,k}$ is the graph consisting of three induced paths of lengths i, j and k, with a common initial vertex.

Canonical tree-decompositions, k-blocks and tangles

REINHARD DIESTEL

Universität Hamburg, Fachbereich Mathematik

Ever since the first graph was decomposed into its blocks by the now well-known block/cutvertex tree, people have wondered how this might be generalized to higher k: can we identify, somehow, 'the k-connected pieces' in a $(k-1)$-connected graph, and decompose it in a tree-like way into those pieces?

Tutte did this for $k = 3$, and some attempts were made for $k = 4$. But there the problem rested for a long time.

In the 1990s, Robertson and Seymour suggested the concept of a *tangle* (of order k) as a notion of a 'k-connected piece' of a graph, and proved that every graph has a tree-decomposition that 'distinguishes' all its maximal tangles. Unlike Tutte's decomposition for $k = 3$, these were not *canonical* in the sense that they depend only on the structure of the graph, but depended on some fixed vertex enumeration.

In the 1970s, Mader had suggested the notion of a k-*block* of a graph: a maximal set of (at least k) vertices no two of which can be separated by $< k$ vertices. Tutte's theorem can be elegantly rephrased by saying that every 2-connected graph has a canonical tree-decomposition that distinguishes all its 3-blocks.

We can now prove this for arbitrary k, and graphs of any connectivity: every finite graph has a canonical tree-decomposition that distinguishes all its k-blocks. In fact, the decomposition can be chosen so as to distinguish all the graph's tangles of order k as well, strengthening the Robertson-Seymour result.

For matroids we have a similar theorem: every finite matroid has a canonical tree-decomposition, one that is invariant under its automorphisms, which distinguishes all its maximal tangles. This strengthens a recent result of Geelen, Gerards and Whittle (who proved this with non-canonical tree-decompositions).

The notion of a k-block suggests many new questions. For example: if $\beta(G)$ denotes the largest k such that G has a k-block, how does β interact with other graph invariants? What natural assumptions will force G to have a k-block? How long will it take to find all the k-blocks of G, or to decide whether one exists?

The total domination polynomial

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A vertex subset $D \subseteq V$ of the graph $G = (V, E)$ is a total dominating set if every vertex of the graph is adjacent to at least one vertex in D. Then the total domination polynomial is the ordinary generating function for the number of total dominating sets in graphs. It is strongly related to the domination polynomial introduced by Arocha and Lliano. In this talk the connection between the total domination polynomial, a generalization of this polynomial and other graph polynomials will be presented. Additionally, for the total domination polynomial some recurrence equations will be given.

Local structure of planar hypohamiltonian graphs

IGOR FABRICI

P.J. Šafárik University, Košice, Slovakia

A graph G is *hypohamiltonian* if it is not hamiltonian but every vertex-deleted subgraph $G - v$ is hamiltonian. In 1978, Thomassen proved that every planar hypohamiltonian graph contains a vertex of degree 3. In this talk we show that every planar hypohamiltonian graph contains an edge of weight (i.e. the sum of degrees of its endvertices) at most 9.

Lifting clique tree inequalities for the quadratic traveling salesman problem

ANJA FISCHER

Technische Universität Dortmund, Fakultät für Mathematik

In this talk we present a new lifting approach for strengthening arbitrary clique tree inequalities of the traveling salesman problem (TSP) that allows us to get stronger valid inequalities for the quadratic TSP. The proof of the main result, showing the validity of the new inequalities, is based on graph-theoretic arguments. Applying the new approach to the well-known subtour elimination constraints leads to new classes of facet defining inequalities of the symmetric QTSP (SQTSP) and the asymmetric QTSP (AQTSP). In the special case of a subtour elimination constraint with exactly two nodes we derive all known conflicting edges/arcs inequalities for the SQTSP and the AQTSP, respectively. These constraints also include valid inequalities of the AQTSP that are not coefficient-symmetric.

This is joint work with Frank Fischer (TU Chemnitz).

Optimisation of a Matroid with one Additional Quadratic Monomial

FRANK FISCHER

Technische Universität Chemnitz, Fakultät für Mathematik

The complete description of the polytope associated with the incidence vectors of the independent sets of a matroid is well-known. Edmonds showed in 1971 that the non-negativity constraints and the rank-inequalities together form a complete description. Recently, Buchheim and Klein suggested to investigate the polytopes associated with polynomially solvable problems that are extended by the linearisation of a single quadratic monomial. The motivation is that these investigations may lead to a better understanding of the polytope of general quadratic optimisation problems. We present a complete description of the polytope associated to the optimisation problem over a matroid extended by one quadratic monomial. Furthermore we fully characterise the facet structure of such polytopes. In particular, this extends and generalises a recent result by Buchheim and Klein [Combinatorial optimization with one quadratic term: Spanning trees and forests, 2013, submitted] concerning the com plete description for the case of the graphical matroid.

This is joint work with Anja Fischer (Dortmund)

Fully Packed Loop configurations on squares and triangles

ILSE FISCHER

Universität Wien

Fully Packed Loop configurations (FPLs) are subgraphs of the square grid such that each internal node is of degree two. While these objects arise naturally in a statistical physics context as a model for ice, they also lead to intriguing enumeration problems. I will start by giving a survey on FPLs of square shape, including Wieland's gyration, the Razumov-Stroganov (ex-)Conjecture and a connection to rhombus tilings. Fully Packed Loop configurations of triangular shape (TFPLs) first appeared in the study of FPLs on a square where they were used to show that the number of FPLs with a given link pattern that has m nested arches is a polynomial function in m. It soon turned out that TFPLs possess a number of other nice properties. For instance, we will see how a curious inequality involving the boundary condition of TFPLs led to the discovery that TFPLs can be seen as a generalized model for Littlewood-Richardson coefficients, thereby establishing an unexpected link to algebra. Moreover, it will be discussed why this could be the starting point for a new approach towards the enumeration of TFPLs.

Another ham sandwich in the plane

ALEXEY GARBER

Moscow State University and Delone Laboratory of Yaroslavl State University

A famous ham sandwich theorem claims that for any d "nice" measures we can find a hyperplane that slices all measures into equal parts. Recently Rade Živaljević proved that it is possible to cut a half of every one of d nice measures in $\mathbb{R}^d$ by a union of several cones of a simple fan translated by some vector. In particularly, this theorem for two-dimensional plane with two measures μ_1 and μ_2 claims that for every 3-fan there is a translation of some cone of this fan that divides both nice measures μ_1 and μ_2 into equal parts.
In this talk we will discuss several generalizations of Živaljević's theorem on the Euclidean plane. We show that every two nice measures in the plane can be partitioned into equal halves by translation of an angle from arbitrary k-fan when k is odd and in some cases when k is even. We also give some counterexamples for certain fans and measures.

This is a joint work with Alexey Balitsky and Roman Karasev.

On fractional f-colourings of vertex-weighted multigraphs

STEFAN GLOCK

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We are urged by the question how similar the f-colouring problem is to the classic edge-colouring problem, particularly with regard to graph parameters. In 2010, Zhang, Yu, and Liu gave a new description of the f-matching polytope and obtained a formula for the fractional f-chromatic index, stating that the fractional f-chromatic index equals the maximum of the fractional maximum f-degree and the fractional f-density. Unfortunately, this formula is incorrect. We present counterexamples for both the description of the f-matching polytope and the formula for the fractional f-chromatic index. Finally, we prove a short lemma concerning the generalization of Goldberg's Conjecture.

Maximum rectilinear crossing number in drawings of the complete graph with a given convex hull

HEIKO HARBORTH

Diskrete Mathematik, TU Braunschweig

Consider n points in the plane where n-h points are in the interor of a convex h-gon. The n points are chosen in such a way that the pairwise connections by straight line segments have at most one point in common either an endpoint or a crossing. Then the maximum number of crossings is determined.

(Common work with Christian Thürmann.)

Grid Intersection Graphs and Dimension

UDO HOFFMANN

Technische Universität Berlin, Institut für Mathematik

Grid intersection graphs (GIGs) are intersection graphs of axis aligned segments, such that no pair of horizontal and no pair of vertical segments intersect. Since they are bipartite they are comparability graphs of partial orders of height 2. We separate several subclasses of GIGs, such as unit GIGs, DORGs, stick and hook graphs, with emphasis on the order dimension.

Joint work with Steven Chaplick, Stefan Felsner and Veit Wiechert.

On maximum weight of a planar graph of given order and size

MIRKO HORŇÁK
(with Andrej Gajdoš, Peter Hudák, Tomáš Madaras)

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The weight of an edge xy of a graph is the sum of degrees of the vertices x and y. The weight of a graph G is the minimum of weights of edges of G. We prove that the weight of any planar graph of order n and size m satisfying $m > 2n$ is smaller than $\frac{9m-12n}{m-2n}$.

Rainbow Numbers for Cycles in Plane Triangulations

Mirko Horňák[1], **STANISLAV JENDROL'**[1], Ingo Schiermeyer[2] and Roman Soták[1]

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Let $\mathcal{T}_n$ be the family of all plane triangulations on n vertices and C_k be a cycle on k vertices. In our talk the existence of rainbow cycles in edge coloured plane triangulations is studied. It is shown that the minimum number $\mathrm{rb}(\mathcal{T}_n, C_3)$ of colours that force the existence of a rainbow C_3 in any n-vertex plane triangulation is equal to $\lfloor \frac{3n-4}{2} \rfloor$. For $k \geq 4$ a lower bound and for $k \in \{4, 5\}$ an upper bound of the number $\mathrm{rb}(\mathcal{T}_n, C_k)$ is determined.

3-colorability of planar graph

LIGANG JIN

Universität Paderborn

A graph $G = (V, E)$ is $(d_1, d_2, \cdots, d_k)$-colorable if V can be partitioned into k subsets $V_1, V_2, \cdots, V_k$ such that $\Delta(G[V_i]) \leq d_i$ for $i = 1, 2, \cdots, k$. The Steinberg Conjecture states that every planar graph with cycles of length neither 4 nor 5 is 3-colorable (i.e. (0,0,0)-colorable). Towards this conjecture, it is known that every planar graph without cycles of length from 4 to 7 is 3-colorable [O. V. Borodin, etc., 2005].
In this talking, we prove every planar graph without cycles of length from 4 to 6 is (1,0,0)-colorable.

This is joint work with Yingli Kang and Yingqian Wang.

A characterization of substar graphs

FELIX JOOS

Universität Ulm

The intersection graphs of stars in some tree are known as substar graphs. We give a characterization of substar graphs by the list of minimal forbidden induced subgraphs. This corrects a flaw in the main result of Chang, Jacobson, Monma and West (Subtree and substar intersection numbers, Discrete Appl. Math. 44, 205-220 (1993)).

Colouring quadrangulations of projective spaces

TOMÁŠ KAISER

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By a well-known result of Youngs, every quadrangulation of the projective plane has chromatic number either 2 or 4. We extend the definition of a quadrangulation to higher dimensions and show that every non-bipartite quadrangulation of the projective n-space has chromatic number at least n+2. We also discuss relations between projective quadrangulations and the classes of Mycielski and Kneser graphs.

The talk is based on joint work with Matej Stehlik.

On the Numbers of Edges of a Fan-Crossing Free Graph

HEUNA KIM

Freie Universität Berlin

A graph drawn in the plane with n vertices is fan-crossing free if there is no triple of edges e, f, and g, such that e and f have a common endpoint and g crosses both e and f. Fan-crossing free graphs arise, for instance, in graph drawing. Humans can read graph drawings in which edges cross at right angles well. Unfortunately, there is previous research showing that only small graphs can be drawn this way: A straight-line drawing of a graph using only right-angle crossings has at most $4n - 10$ edges. Since such a graph is fan-crossing free, this led to the question: "what is the maximum number of edges of a fan-crossing free graph on n vertices?"

We answer this question by showing that a fan-crossing free graph has at most $4n - 8$ edges (and at most $4n - 9$ edges with straight-line drawings). We generalize our result to graphs without radial $(k, 1)$-grids; that is, sets of k edges all incident to a common endpoint that are all crossed by another edge e.
Finally, we give a very general bound for a monotone graph property.

Plane Cubic Graphs and the Air-Pressure Method

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Thomassen proved that plane cubic graphs are area-universal, that is, for a plane cubic graph G with positive prescribed face areas there exists a straight-line redrawing G' that realizes these areas. Thomassen uses induction and proves the existence of a degenerate drawing. We show that plane cubic graphs are area-universal using the air-pressure method. The name-giving property of the method is to define the pressure of a face as the fraction of its prescribed and its current area. These pressures exert forces on the vertices and the idea is to move the inner vertices along the forces such that the prescribed areas are realized. A similar method has previously been applied in the context of area-universality of rectangular layouts. Within the talk, we show how the method can be adapted for other classes of plane graphs, in particular for plane cubic graphs.

This is joint work with Stefan Felsner.

Extending the MAX Algorithm for Maximum Independent Set

NGOC LE, Ingo Schiemeyer and Christoph Brause

Faculty of Mathematics and Informatics, Technical University of Freiberg

The Maximum Independent Set problem is an NP-hard problem. In this talk we consider the algorithm MAX, which is a polynomial time algorithm for finding a maximal independent set in a graph G. We present a set of forbidden induced subgraphs such that the algorithm MAX always results in finding a maximum independent set of G. We also describe a modification of the MAX algorithm and a set of forbidden induced subgraphs for the new algorithm.

Relating Ordinary and Total Domination in Cubic Graphs of Large Girth

Christian Löwenstein

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For a cubic graph G of order n, girth at least g, and domination number $\left(\frac{1}{4}+\epsilon\right)n$ for some $\epsilon \geq 0$, we show that the total domination number of G is at most $\frac{13}{32}n + O\left(\frac{n}{g}\right) + O(\epsilon n)$, which implies $\frac{\gamma_t(G)}{\gamma(G)} \leq 1.89714 + O\left(\frac{n}{g}\right)$.

$(\mathcal{P}, \mathcal{Q})$-Total (r, s)-Colorings of Graphs

MASSIMILIANO MARANGIO

Computational Mathematics, Technische Universität Braunschweig

Let $r, s \in \mathbb{N}$, $r \geq s$, and $\mathcal{P}$ and $\mathcal{Q}$ be two additive and hereditary graph properties. A $(\mathcal{P}, \mathcal{Q})$-total (r, s)-coloring of a graph $G = (V, E)$ is a coloring of the vertices and edges of G by s-element subsets of $\mathbb{Z}_r$ such that for each color i, $0 \leq i \leq r-1$, the vertices colored by subsets containing i induce a subgraph of G with property $\mathcal{P}$, the edges colored by subsets containing i induce a subgraph of G with property $\mathcal{Q}$, and color sets of incident vertices and edges are disjoint. The fractional $(\mathcal{P}, \mathcal{Q})$-total chromatic number $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$ of G is defined as the infimum of all ratios r/s such that G has a $(\mathcal{P}, \mathcal{Q})$-total (r, s)-coloring.
In this talk general lower and upper bounds for $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$ are presented as well as some exact values for specific properties and specific classes of graphs.

This is joint work with Arnfried Kemnitz, Anja Pruchnewski, and Margit Voigt.

The Query Complexity of Finding a Hidden Permutation

KURT MEHLHORN

Max-Planck-Institut für Informatik

We study the query complexity of determining a hidden permutation. More specifically, we study the problem of learning a secret (z, π) consisting of a binary string z of length n and a permutation π of $[n]$. The secret must be unveiled by asking queries $x \in \{0,1\}^n$, and for each query asked, we are returned the score $f_{z,\pi}(x)$ defined as

$$f_{z,\pi}(x) := \max\{i \in [0..n] \mid \forall j \leq i : z_{\pi(j)} = x_{\pi(j)}\} ;$$

i.e., the length of the longest common prefix of x and z with respect to π. The goal is to minimize the number of queries asked. We prove matching upper and lower bounds for the deterministic and randomized query complexity of $\Theta(n \log n)$ and $\Theta(n \log \log n)$, respectively.

Joint work with Peyman Afshani, Manindra Agrawal, Benjamin Doerr, Carola Doerr, and Kasper Green Larsen

Cycles in Complementary Prisms

DIRK MEIERLING

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The complementary prism $G\overline{G}$ of a graph G arises from the disjoint union of G and the complement $\overline{G}$ of G by adding a perfect matching joining corresponding pairs of vertices in G and $\overline{G}$. Partially answering a question posed by Haynes et al. (The complementary product of two graphs, Bull. Inst. Comb. Appl. 51, 21-30, 2007) we provide an efficient characterization of the circumference of the complementary prism $T\overline{T}$ of a tree T and show that $T\overline{T}$ has cycles of all lengths between 3 and its circumference. Using a similar argument, we also characterize those bipartite graphs whose complimentary prism is Hamiltonian. Finally, we prove that for a given graph of bounded maximum degree it can be decided in polynomial time whether its complementary prism is Hamiltonian.

Notes on Exchange Stable Matchings

Andrei Asinowski, Balázs Keszegh, **TILLMANN MILTZOW**

Freie Universität Berlin

Given two sets A, B with $|A| = m$ and $|B| = n$. We assume that every element $a \in A$ has a priority list of B. We call an injective mapping τ from A to B a matching. Such a matching τ is *not* exchange stable if there exists another matching τ' such that at least some element in A improves in τ' w.r.t. its preference list and no one gets worse. An element $b \in B$ is selectable if there exists an exchange stable matching using b. The set of all selectable elements are denoted by $E(M)$. We show

$$|E(M)| \leq \sum_{i=1,\ldots,m} \left\lfloor \frac{m}{i} \right\rfloor = \Theta(m \log m).$$

And this is asymptotically tight.

On orthogonal ray trees

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Orthogonal ray graphs (ORGs) are the bipartite intersection graphs of orthogonal half-lines in the plane, where the partitions correspond to vertical, and horizontal rays respectively, and form a subclass of (unit) grid intersection graphs. While the problem of their recognition is in general still open, the ORG trees admit several equivalent characterizations, out of which a polynomial recognition algorithm is derived. We relate the problem to the description of 2-directional orthogonal ray trees based on forbidden subgraphs, and show the equivalence of our result to that of Nishikawa, Tayu and Ueno, based on forbidding asteroidal quintuples.

The r-generalization of combinatorial numbers

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A. Z. Broder defined a certain generalization of Stirling numbers, the so-called r-Stirling numbers of the first and second kind. They were later rediscovered by R. Merris, while I. Mező introduced r-Bell numbers. In our talk we overview our results about the r-generalization of further combinatorial numbers like Lah numbers, Fubini numbers and Eulerian numbers.

These results are joint with István Mező and Gabriella Rácz.

Parking functions for trees and mappings

ALOIS PANHOLZER

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The notion of parking functions has been introduced in the 60's during the analysis of hashing-algorithms. Since then parking functions are an important and intensely studied concept in combinatorics with various generalizations. Here we introduce parking functions for rooted trees and n-mappings, i.e., functions from the set $[n] := \{1, 2, \ldots, n\}$ into itself. For trees the setting is as follows: Given a rooted labelled tree with n nodes, m drivers successively try to park at their specific preferred parking space. If this node is already occupied, the driver moves on to the next node lying closer to the root. How many assignments of preferred parking spaces are there that allow all drivers to park? In order to answer this question both with exact and asymptotic enumeration results we carry out an analytic combinatorics treatment of the problem, which naturally requires a study of certain linear and quasi-linear PDEs.

Proof of a conjecture on difference balanced functions

ALEXANDER POTT and Qi Wang

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Difference balanced functions defined on a finite field are closely related to combinatorial designs and naturally define p-ary sequences with ideal two-level autocorrelation. All such functions are d-homogeneous, and Gong and Song conjectured that all difference balanced functions must have this property.

We discuss an equivalence between the d-homogeneous property and multipliers of generalized difference sets. By determining these multipliers, we prove the Gong-Song conjecture for fields of prime order.

The Erdős Pósa Property of Long Cycles

THOMAS SASSE

Universität Ulm

For an integer ℓ at least 3, we prove that if G is a graph containing no two vertex-disjoint circuits of length at least ℓ, then there is a set X of at most $\frac{5}{3}\ell + \frac{29}{2}$ vertices that intersects all circuits of length at least ℓ. Our result improves the bound $2\ell + 3$ due to Birmelé, Bondy, and Reed (The Erdős-Pósa property for long circuits, Combinatorica 27 (2007), 135-145) who conjecture that ℓ vertices always suffice.

Rainbow connection and size of graphs

INGO SCHIERMEYER

Technische Universität Bergakademie Freiberg

An edge-coloured connected graph G is called *rainbow-connected* if each pair of distinct vertices of G is connected by a path whose edges have distinct colours. The *rainbow connection number* of G, denoted by $rc(G)$, is the minimum number of colours such that G is rainbow-connected. In this talk we consider the following problem:

Problem For all integers n and k with $1 \leq k \leq n-1$ compute and minimize the function $f(n,k)$ with the following porperty: If $|V(G)| = n$ and $|E(G)| \geq f(n,k)$ then $rc(G) \leq k$.

For n and k with $1 \leq k \leq n-1$ it holds that $f(n,k) \geq \binom{n-k+1}{2} + k - 1$. It has been shown that $f(n,k) = \binom{n-k+1}{2} + k - 1$ for $k = 1, 2, 3, 4$ and for $n - 6 \leq k \leq n - 1$.
In this talk we will report about these results and show some further recent progress obtained for this problem.

Mondshein's Legacy

JENS M. SCHMIDT

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We study a long-forgotten generalization of canonical orderings to non-planar graphs that was published by Lee Mondshein in a PhD-thesis at M.I.T. as early as 1971. Mondshein proposed to order the vertices of a graph in a sequence such that, for any i, the vertices from 1 to i induce essentially a 2-connected graph while the remaining vertices from $i+1$ to n induce a connected graph.
We present the first algorithm that computes a Mondshein sequence in time and space $O(m)$, improving the previous best running time by a factor of n. From this result, we deduce linear-time algorithms for several other problems, for which the previous best running times have been quadratic.

Circular flows on signed graphs

MICHAEL SCHUBERT
(JOINT WORK WITH ECKHARD STEFFEN)

Universität Paderborn

Bouchet's conjecture, that every flow-admissible signed graph admits a nowhere-zero 6-flow is equivalent to its restriction on cubic graphs. We give a proof of Bouchet's conjecture for Kotzig-graphs.
We show that there exist signed graphs where the difference between the integer flow number and the flow number is greater than or equal to 1, disproving a conjecture of Raspaud and Zhu. In order to generalize Tutte's characterization of bipartite cubic graphs, we introduce the notion of r-minimal sets for $r \geq 2$. A set $X \subseteq E(G)$ is a minimal set such that $G - X$ is bipartite if and only if X is $(2 + \frac{1}{t})$-minimal. Furthermore, $2 + \frac{1}{t}$ is an element of the flow spectrum $\mathcal{S}(G)$ of a $(2t+1)$-regular graph G if and only if G has a t-factor. If G has a 1-factor, then $3 \in \overline{\mathcal{S}}(G)$, and for every $t \geq 2$, there is a signed $(2t+1)$-regular graph (H, σ) where 3 is an element of its integer flow spectrum and H does not have a 1-factor.

Constructive bounds for the facial Thue choice number

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There are several ways how to prove some graph properties in discrete mathematics. One can distinguish between constructive and non-constructive approaches. By proving some general upper bounds for a list colouring invariant of graphs one uses more often a non-constructive proof which showes the existence of a particular kind of object without providing an example. By knowing a little bit more about the structure of a graph also a constructive proof can give an upper bound for some list colouring graph invariant.
Here we show some constructive upper bounds for the facial Thue choice number of graphs - one of the Thue type colouring invariants of graphs.

Exclusive sum labelings of hypergraphs

MARTIN SONNTAG

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(joint work with HANNS-MARTIN TEICHERT, Universität zu Lübeck)

We generalize the concept of exclusive sum labelings of graphs (cf. M. MILLER, D. PATEL, J. RYAN, K.A. SUGENG, SLAMIN AND M. TUGA: *Exclusive sum labeling of graphs*; JCMCC **55** (2005), 137-148) to hypergraphs and determine the exclusive sum number for several classes of hypergraphs.
The results disclose a significant difference between graphs (i.e. 2-uniform hypergraphs) and "genuine" hypergraphs.

Star Edge Coloring of Trees and Outerplanar Graphs

ROMAN SOTÁK

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Let $\varphi : E \to \{1, 2, \ldots, k\}$ be a proper edge coloring of a graph G. A coloring φ is a *star edge coloring* if there is no bi-colored 4-cycle and no bicolored path of lenght 4 in G. The *star chromatic index* of G, denoted by $\chi'_s(G)$, is the minimum number of colors required for such coloring.
Recently, Dvořák, Mohar and Šámal [J. Graph Theory, 2013] obtained a near-linear upper bound on χ'_s in terms of the maximum vertex degree of graph. They also provided some bounds for complete graphs and subcubic graphs.
We present some results on the star edge coloring of outerplanar graphs; we determine the best possible upper bound on the star chromatic index of trees and subcubic outerplanar graphs. We also derive an upper bound for outerplanar graphs, which is best possible up to constant.

Circular flows and edge-colorings on regular graphs

ECKHARD STEFFEN

Universität Paderborn

We generalize Tutte's characterizations of cubic graphs with flow number 3 and of cubic graphs with flow number 4 to $(2t + 1)$-regular graphs. It turns out that the class of cubic graphs is the only class of odd regular graphs where a flow number separates the class 1 graphs from the class 2 graphs.
We further generalize some of the above results to flows on signed $(2t + 1)$-regular graphs.

On the Number of Dominating Sets

PETER TITTMANN

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We proved in [2] that the number $d(G)$ of dominating sets of an undirected graph G can be represented as sum over vertex induced subgraphs of G such that all components of each induced subgraph are of even order. Here we give further representations of $d(G)$ and present some new proofs for the fact (see [1]) that the number of dominating sets of any graph is odd.

References

[1] Andries E. Brouwer: The number of dominating sets of a finite graph is odd, preprint, (2009) http://www.win.tue.nl/~aeb/preprints/domin2.pdf.

[2] Tomer Kotek, James Preen, Peter Tittmann: Subset-Sum Representations of Domination Polynomials. Graphs and Combinatorics, 2013.

Subgraphs of dense multipartite graphs

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Turán's theorem tells us that every n-vertex graph with at least $t_r(n) + 1$ edges contains a clique of size $r + 1$, where $t_r(n)$ stands for the number of edges of the r-partite Turán graph of order n. The theorem has been strengthened in various ways. Erdős (1963) and Nikiforov (2013) proved the following theorem.

Let $r \geq 2$, and let $K_r^+(s_1, \ldots, s_r)$ be the graph obtained by adding an edge to the first part of the complete r-partite graph $K_r(s_1, \ldots, s_r)$. Then for all sufficiently small $\varepsilon > 0$, every graph of sufficiently large order n with $t_r(n) + 1$ edges contains a copy of $K_r^+(\lfloor c \ln n \rfloor, \ldots, \lfloor c \ln n \rfloor, \lfloor n^{1-\varepsilon} \rfloor)$.
Bollobás and Nikiforov (2008) improved an result of Erdős showing that

If $r \geq 2$ and $n > r^8$, then every n-vertex graph with $t_r(n) + 1$ edges contains a collection of n^{r-1}/r^{r+5} distinct $(r + 1)$-cliques sharing an edge.
In this work we prove analogous results for multipartite graphs using stability method.

This is a joint work with Lothar Narins (FU Berlin).

A new vincular pattern based Mahonian statistic on words

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In 2000 Babson and Steingrímsson re-defined known Mahonian statistics on permutations and defined new ones, their approach being based on vincular pattern involvement; one of these new statistics is STAT. Refining a previous result of the second author of this talk, we show that the natural extension of STAT to words is still Mahonian; and numerical evidences let us believe that it is the unique statistic among those defined by Babson-Steingrímsson with this property. More precisely, we construct an explicit bijection between words with a fixed value for the major index and those with the same value for STAT, and we give some particular properties of both, this bijection and the extension of STAT to words.

[1] This work was partially supported by *La Région de Bourgogne, France*.

The Erdős–Szekeres Theorem and Related Results

PAVEL VALTR

The Erdős–Szekeres k-gon theorem (1935) says that for any integer $k \geq 3$ there is an integer $n(k)$ such that any set of $n(k)$ points in the plane, no three on a line, contains k points which are vertices of a convex k-gon. It is a classical result both in combinatorial geometry and in Ramsey theory.
We will discuss various results and open problems related to the Erdős–Szekeres theorem. For example, it is still widely open if the minimum possible value of $n(k)$ is equal to $2^{k-2} + 1$, as conjectured by Erdős and Szekeres more than fifty years ago. It might be particularly interesting for this audience that some recent results related to the Erdős–Szekeres theorem are purely combinatorial, dealing with colored (hyper)graphs on linearly ordered vertex sets.

Some inequalities on spindle convex discs

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Spindle convex discs are convex discs that can be written as the intersection of (not neccesarily finitely many) closed unit circles. Well-known examples are Reuleux-polygons. We give some new inequalities regarding spindle convex discs in spirit of the well-known Blaschke-Santaló inequality and the Blaschke-Lebesgue theorem.

Balancing Pairs for Perfect Elimination Orderings

VEIT WIECHERT

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One of the most intriguing problems for posets is the balanced pair conjecture (also known as $\frac{1}{3}$-$\frac{2}{3}$ conjecture). It states, that every finite poset has an incomparable pair (x, y) such that the probability of x lying before y in a randomly chosen linear extension is between 1/3 and 2/3. Recently, Eppstein proposed a generalization, concerning balancing pairs for basic words of antimatroids. We study the case, where the basic words correspond to perfect elimination orderings (PEOs) of chordal graphs and show that the conjecture is true for PEOs of unit interval graphs.

Joint work with Stefan Felsner.